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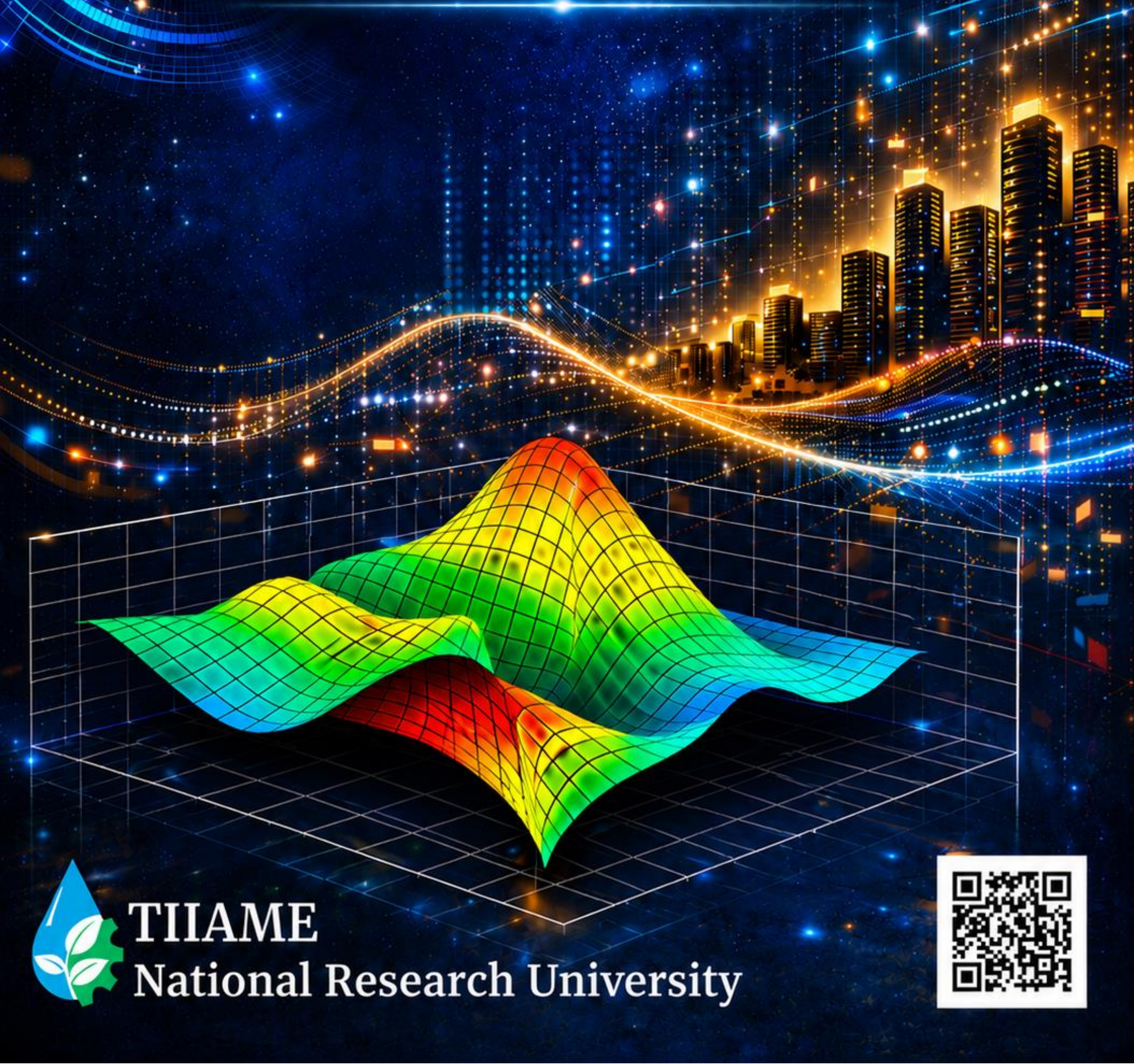

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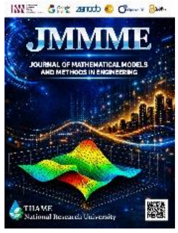

Effect of support conditions on nonlinear parametric vibrations of a viscoelastic anisotropic reinforced composite plate

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ABSTRACT

This study considers the nonlinear dynamics of a rectangular reinforced plate made of glass-reinforced plastic subjected to a periodic compressive load along one pair of edges. The model is based on a refined Timoshenko-type theory, which retains transverse shear and rotary inertia, and it includes geometric nonlinearity together with Boltzmann–Volterra hereditary viscoelasticity governed by the weakly singular Koltunov–Rzhanitsyn kernel. The anisotropy of the structure is described by transformed reduced stiffnesses at an arbitrary reinforcement angle. Applying the Bubnov–Galerkin method reduces the problem to a system of nonlinear integro-differential equations, which are integrated numerically once the singularity of the kernel has been removed. Two sets of boundary conditions are considered: simple support and clamping of all edges, with the material, geometry, rheological parameters, and loading law kept identical, which makes it possible to attribute the difference in the results to the type of support. The sensitivity of the response to the reinforcement angle is shown to be governed by the support conditions: for simple support, rotating the fibers from 0° to 45° reduces the deflection at the midpoint by roughly a factor of three, whereas for clamping the same rotation changes the amplitude by about 10% and in the opposite direction. Taking hereditary properties into account turns undamped vibrations into a decaying process without external damping, in the same way for both types of support. The clamped plate is found to require nine retained modes compared with four for simple support, and the reason for the difference is identified.

1. Introduction

Laminated polymer composites are used in aircraft skins and panels, in blades, and in hull and span structures. Their advantage over metals lies not in stiffness as such, but in the ability to control it through the fiber layup and in the energy dissipated by the polymer matrix. Analyzing such structures requires two properties to be taken into account that are usually ignored for metal structures. Anisotropy is determined by the reinforcement angle and by the stacking sequence, so the laminate stiffnesses depend on direction. Creep and stress relaxation develop in the polymer matrix. Under cyclic loading, part of the energy is dissipated inside the material itself, and this

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dissipation is comparable in magnitude to the external damping of the structure.

The way hereditary properties are taken into account determines both the formulation of the problem and its outcome. The Voigt model gives internal friction proportional to frequency, which is not confirmed experimentally: for glass-reinforced plastics, friction over a wide frequency range is almost independent of the strain rate [1, 2]. The Boltzmann–Volterra hereditary model imposes no such restriction, while the Koltunov–Rzhanitsyn kernel, with its weak singularity at $t = 0$, describes the initial stage of relaxation, where the discrepancy with an exponential kernel is most pronounced; its parameters for glass-reinforced plastics have been determined experimentally [3].

Under a periodic compressive load, stability is lost not when the static critical value is reached, but when the excitation frequency falls into a region of dynamic instability, where the amplitude grows even at loads well below the Euler load. The theory of such problems was developed by Bolotin [4], who reduced the equations of motion to a Mathieu–Hill equation; the nonlinear effects of parametric resonance were systematized by Nayfeh and Mook [5], and refined theories of laminated plates are presented by Reddy [6], Qatu [7], and Whitney [8].

Recent work has made the kinematic model more complex and widened the class of materials treated. Darabi and Ganesan [9] constructed the instability boundaries of laminated cylindrical shells using Bolotin's method, Sheng and Wang [10] studied shells made of a functionally graded material under combined parametric and external excitation, Fu et al. [11] extended the analysis to porous conical shells in a thermal environment, and Chen et al. [12] showed that the choice of the kinematic hypothesis shifts the boundaries of these regions. Jain et al. [13] found that the edge support changes the width of the instability zone at least as much as the loading parameters do, yet different support schemes have not been compared systematically.

All of this work shares a common scheme: the problem is reduced to a single second-order equation with periodic coefficients, after which the instability boundaries are constructed using Bolotin's method. The scheme is economical, but it assumes retention of a single mode and linearization, and it introduces dissipation through an external coefficient or through a viscous model, which is inaccurate for polymer composites, where the stress is governed by the entire deformation history.

For viscoelastic systems an integral operator inevitably appears in the equations of motion, and after Bubnov–Galerkin reduction the problem becomes a system of nonlinear integro-differential equations with weakly singular kernels. A stable method of solving them, based on quadrature formulas and on a change of variables that removes the singularity of the kernel, was proposed by Badalov, Eshmatov, and Yusupov [14] and developed further by Verlan, Abdikarimov, and Eshmatov [15]. On this basis a number of problems in the dynamics of viscoelastic thin-walled structures have been solved [16, 17, 18, 19]: accounting for hereditary properties lowers the calculated critical loads and turns the undamped vibrations of the elastic problem into a decaying process.

Within this line of research, the effect of the support conditions remains the least studied. Simple support admits a trigonometric approximation that satisfies the boundary conditions exactly, and for this reason it is used in most calculations; clamping requires different coordinate functions and a larger number of retained modes, and results for clamped plates are reported less often. No direct comparison of the two types of support for identical material, geometry, rheology, and loading law is available in the literature known to the authors. Yet the trends established for simple support are carried over to clamped plates as if they were self-evident. This applies above all to the effect of the reinforcement angle: choosing the layup is a recognized way of lowering the vibration level, but whether the effect survives under clamping remains an open question. The answer has a direct design consequence: if the sensitivity to the reinforcement angle depends on how the edges are supported, then a layup optimization carried out for one support scheme loses its meaning when another is used.

The aim of this work is to carry out such a comparison. Within a single model that includes transverse shear and rotary inertia, von Kármán geometric nonlinearity, laminate anisotropy at an arbitrary reinforcement angle, and hereditary viscoelasticity with the Koltunov–Rzhanitsyn kernel, the behavior of a reinforced plate made of glass-reinforced plastic under a periodic compressive load is calculated. For simple support and for clamping of all edges, convergence with respect to the number of retained modes is examined, and the time histories of the deflection at the midpoint are obtained as the aspect ratio, the thickness, the reinforcement angle, and the frequency of the external load are varied.

The novelty of the work lies in three results. It is shown that the sensitivity of the response to the reinforcement angle is governed by the support conditions: for simple support, rotating the fibers from 0° to 45° reduces the deflection at the midpoint by roughly a factor of three, while for clamping it changes the amplitude by about 10%. The difference is found to persist when hereditary properties are included, because the integral operator multiplies all reduced stiffnesses by the same factor and does not redistribute them between directions. It is also shown that for clamping, convergence is reached with nine retained modes compared with four for simple support, and the reason for the difference is identified.

2. Materials and methods

Consider a rectangular plate with sides a and b, of constant thickness h, composed of K unidirectionally reinforced layers. The fibers of each layer make an angle θ with the Ox axis. Compressive forces are applied to the edges $x = 0$ and $x = a$; these forces vary in time according to the law

$$P(t) = P_0 + P_t \cos\gamma t \tag{1}$$

where P_0 denotes the constant part of the load, P_t the amplitude of the variable part, and γ the frequency of the external periodic excitation. The analysis model is shown in Fig. 1.

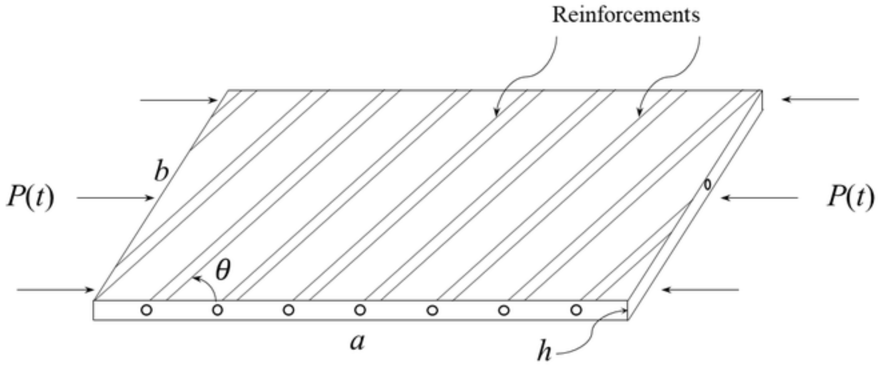


Fig. 1. Reinforced plate under a periodic compressive load

Two limiting cases of edge support are considered: simple support along the entire contour and clamping along the entire contour. All other parameters of the problem are the same in both cases, and this is what makes it possible to attribute the difference in the results to the support conditions rather than to the formulation.

The constitutive relations are based on a refined Timoshenko-type theory, which retains transverse shear strains and rotary inertia [6, 7, 8]. The normal and shear forces N_x, N_y, T, the bending and twisting moments M_x, M_y, H, and the transverse shear forces Q_x, Q_y are related to the midsurface strains and to the rotations of the normal ψ_x, ψ_y by the relations

$$\begin{Bmatrix} N_x \\ N_y \\ T \\ M_x \\ M_y \\ H \\ Q_y \\ Q_x \end{Bmatrix} = \begin{bmatrix} A_{11}^* & A_{12}^* & A_{16}^* & B_{11}^* & B_{12}^* & B_{16}^* & 0 & 0 \\ A_{12}^* & A_{22}^* & A_{26}^* & B_{12}^* & B_{22}^* & B_{26}^* & 0 & 0 \\ A_{16}^* & A_{26}^* & A_{66}^* & B_{16}^* & B_{26}^* & B_{66}^* & 0 & 0 \\ B_{11}^* & B_{12}^* & B_{16}^* & D_{11}^* & D_{12}^* & D_{16}^* & 0 & 0 \\ B_{12}^* & B_{22}^* & B_{26}^* & D_{12}^* & D_{22}^* & D_{26}^* & 0 & 0 \\ B_{16}^* & B_{26}^* & B_{66}^* & D_{16}^* & D_{26}^* & D_{66}^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{44}^* & A_{45}^* \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{45}^* & A_{55}^* \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \partial\psi_x/\partial x \\ \partial\psi_y/\partial y \\ \partial\psi_x/\partial y + \partial\psi_y/\partial x \\ \partial w/\partial y + \psi_y \\ \partial w/\partial x + \psi_x \end{Bmatrix} \tag{2}$$

The matrix in (2) combines the membrane, membrane–bending, bending, and shear characteristics of the laminate.

The operators appearing in (2) are defined by integration through the thickness of the laminate:

$$A_{ij}^{*}\varphi = \sum_{k=1}^{K} Q_{ij}^{*}\,\varphi_k(z_k - z_{k-1}), \qquad B_{ij}^{*}\varphi = \frac{1}{2}\sum_{k=1}^{K} Q_{ij}^{*}\,\varphi_k(z_k^2 - z_{k-1}^2),$$
$$D_{ij}^{*}\varphi = \frac{1}{3}\sum_{k=1}^{K} Q_{ij}^{*}\,\varphi_k(z_k^3 - z_{k-1}^3), \quad i,j = 1,2,6; \qquad A_{ij}^{*}\varphi = K_s\sum_{k=1}^{K} Q_{ij}^{*}\,\varphi_k(z_k - z_{k-1}), \quad i,j = 4,5 \tag{3}$$

where z_k is the coordinate of the upper boundary of the kth layer and K_s is the shear correction factor. The calculations use $K_s = 5/6$, the value adopted in the refined plate theory of Ambartsumyan and Reissner. The overall formulation of the problem follows [19, 20, 21].

The reduced stiffnesses of a layer rotated through an angle θ about the Ox axis are written with the hereditary integral operator included:

$$\begin{aligned}
Q_{11}^{*}\varphi &= [Q_{11}\cos^4\theta + 1/2\,(Q_{12} + 2Q_{66})\sin^2 2\theta + Q_{22}\sin^4\theta](1 - \Gamma^{*})\varphi,\\
Q_{22}^{*}\varphi &= [Q_{11}\sin^4\theta + 1/2\,(Q_{12} + 2Q_{66})\sin^2 2\theta + Q_{22}\cos^4\theta](1 - \Gamma^{*})\varphi,\\
Q_{12}^{*}\varphi &= [1/4\,(Q_{11} + Q_{22} - 4Q_{66})\sin^2 2\theta + Q_{12}(1 - 1/2\sin^2 2\theta)](1 - \Gamma^{*})\varphi,\\
Q_{66}^{*}\varphi &= [1/4\,(Q_{11} - 2Q_{12} + Q_{22})\sin^2 2\theta + Q_{66}\cos^2 2\theta](1 - \Gamma^{*})\varphi,\\
Q_{16}^{*}\varphi &= [Q_{11}\sin\theta\cos^3\theta - 1/4\,(Q_{12} + 2Q_{66})\sin 4\theta - Q_{22}\sin^3\theta\cos\theta](1 - \Gamma^{*})\varphi,\\
Q_{26}^{*}\varphi &= [Q_{11}\sin^3\theta\cos\theta - 1/4\,(Q_{12} + 2Q_{66})\sin 4\theta - Q_{22}\sin\theta\cos^3\theta](1 - \Gamma^{*})\varphi
\end{aligned} \tag{4}$$

The stiffnesses in the principal axes of elasticity are expressed in terms of the engineering constants:

$$Q_{11} = \frac{E_1}{1-\mu_{12}\mu_{21}}, \quad Q_{22} = \frac{E_2}{1-\mu_{12}\mu_{21}}, \quad Q_{12} = \frac{E_1\mu_{21}}{1-\mu_{12}\mu_{21}} = \frac{E_2\mu_{12}}{1-\mu_{12}\mu_{21}}, \quad Q_{66} = G_{12} \tag{5}$$

where E_1 and E_2 are the elastic moduli along and across the fibers, G_{12} is the shear modulus, and μ_{12} and μ_{21} are Poisson's ratios.

Hereditary properties are taken into account through the Boltzmann–Volterra integral operator

$$\Gamma^{*}\varphi = \int_0^t \Gamma\,(t-\tau)\varphi(\tau)d\tau \tag{6}$$

where $\Gamma(t)$ is the relaxation kernel, t is the observation time, and τ is a preceding instant of time.

The midsurface strains are related to the displacements u, v, and w in the von Kármán sense:

$$\varepsilon_x^0 = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \qquad \varepsilon_y^0 = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2, \qquad \gamma_{xy}^0 = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \tag{7}$$

The quadratic terms in (7) couple bending with stretching of the midsurface and limit the growth of the amplitude at parametric resonance.

Substituting (1), (2), and (7) into the equations of motion gives

$$\begin{aligned}
&\frac{\partial N_x}{\partial x} + \frac{\partial T}{\partial y} + p_x = \rho\frac{\partial^2 u}{\partial t^2}, \qquad \frac{\partial T}{\partial x} + \frac{\partial N_y}{\partial y} + p_y = \rho\frac{\partial^2 v}{\partial t^2},\\
&\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + \frac{\partial}{\partial x}\left(N_x\frac{\partial w}{\partial x} + T\frac{\partial w}{\partial y}\right) + \frac{\partial}{\partial y}\left(T\frac{\partial w}{\partial x} + N_y\frac{\partial w}{\partial y}\right) + P(t)\frac{\partial^2 w}{\partial x^2} + q = \rho\frac{\partial^2 w}{\partial t^2},\\
&\frac{\partial M_x}{\partial x} + \frac{\partial H}{\partial y} - Q_x = \frac{\rho h^2}{12}\frac{\partial^2 \psi_x}{\partial t^2}, \qquad \frac{\partial H}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y = \frac{\rho h^2}{12}\frac{\partial^2 \psi_y}{\partial t^2}
\end{aligned} \tag{8}$$

where ρ is the density of the material, and p_x, p_y, and q are the components of the external surface load. The last two equations in (8) describe the rotary inertia of the normal. The term $P(t)\,\partial^2 w/\partial x^2$ introduces parametric excitation: the coefficient of the second derivative of the deflection depends on time.

System (8) in the five unknowns u, v, w, ψ_x, and ψ_y is a set of nonlinear partial integro-differential equations; the integral operator enters through the stiffnesses (4).

The weakly singular Koltunov–Rzhanitsyn kernel is taken as the relaxation kernel

$$\Gamma(t) = A\,e^{-\beta t}\,t^{\alpha-1}, \qquad 0 < \alpha < 1 \tag{9}$$

with rheological parameters A, α, and β determined from relaxation tests [1, 2]. The calculations use the values $A = 0.0208$, $\alpha = 0.1$, and $\beta = 0.00166$, obtained for glass-reinforced plastics [3].

The solution of system (8) is sought in the form of expansions in coordinate functions that satisfy the boundary conditions of the problem.

The number of retained terms was chosen on the basis of the convergence analysis given in Section 3: for simple support the first four harmonics were retained ($M = N = 2$), and for clamping the first nine ($M = N = 3$).

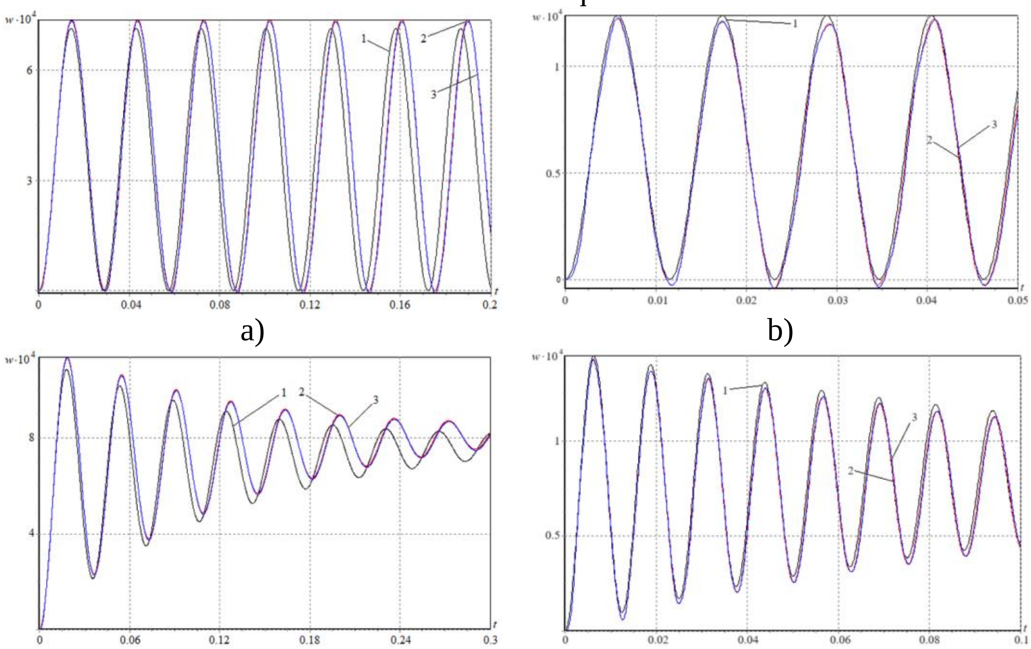
Kernel (9) has a singularity at $t = 0$, so quadrature formulas cannot be applied directly to the integral term. The singularity is removed by a change of the integration variable, after which the kernel becomes regular, and quadrature formulas are then applied to the transformed system following the scheme proposed in [14] and developed in [15]. At each time step this yields a system of algebraic equations, which is solved sequentially. The initial conditions are taken to be zero.

The calculations were carried out for a plate of KAST-V glass-reinforced plastic [3] with the following properties: $E_1 = 25.5$ GPa, $E_2 = 14.91$ GPa, $G_{12} = 4.41$ GPa, $\mu_{12} = 0.2$, $\rho = 1900$ kg/m^3. The baseline geometry is $a = b = 0.5$ m, $h = 0.5$ cm, and the reinforcement angle is $\theta = 45°$. In studying the effect of individual parameters, the aspect ratio $\lambda = a/b$ was varied from 1 to 1.4, the thickness from 0.48 to 0.52 cm, the reinforcement angle from 0 to 45°, and the frequency of the external load γ from 1 to 3; the remaining quantities were kept unchanged.

3. Results

All the curves given below refer to the deflection at the midpoint of the plate. The calculations for simple support and for clamping were carried out as part of the studies [20, 21]; in the present paper they are compared with each other for the first time for identical material, geometry, rheological parameters, and loading law. The deflection is given in meters and the time in seconds. Each figure has two panels: a corresponds to simple support and b to clamping. All other conditions within a given figure are the same, so the difference between the panels is due solely to the way the edges are supported.

Convergence with respect to the number of retained modes was checked separately for the elastic and for the viscoelastic formulation, since the hereditary integral operator changes the nature of the solution and could affect the number of series terms required.



a) b)

c) d)

Fig. 2. Convergence of the Bubnov–Galerkin method: a and b – elastic problem, c and d – viscoelastic problem; a and c – simple support (curves: 1 – $M = N = 1$, 2 – $M = N = 2$, 3 – $M = N = 3$), b and d – clamping (curves: 1 – $M = N = 2$, 2 – $M = N = 3$, 3 – $M = N = 4$)

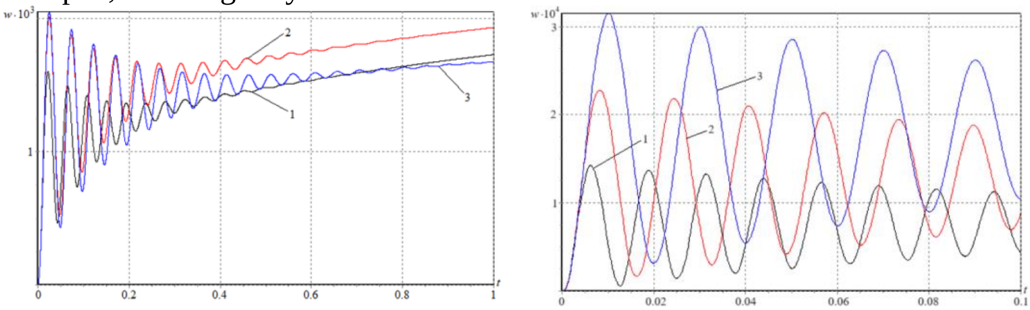
For simple support, retaining the first four harmonics ($M = N = 2$) gives a curve that is virtually indistinguishable from the solution for nine harmonics: the difference in amplitude is smaller than the line thickness. Increasing the number of terms further does not change the result, so the remaining calculations for this type of support use $M = N = 2$.

For clamping, four harmonics are insufficient: the solution for $M = N = 2$ differs noticeably from the solutions for $M = N = 3$ and $M = N = 4$, whereas the latter two are virtually indistinguishable. The remaining calculations for the clamped plate use $M = N = 3$, that is, nine retained modes.

The difference in the number of modes required is explained by the form of the coordinate functions. The trigonometric functions used for simple support coincide with the natural vibration modes of the plate, and the first of them alone carries the main part of the solution. The beam functions used for clamping contain hyperbolic terms whose contribution is concentrated near the edges and decays rapidly toward the interior of the plate, so more series terms are needed to describe the edge zone.

Comparing the panels of Fig. 2 gives a second result, unrelated to convergence. In the elastic problem the deflection curves represent undamped vibrations of constant amplitude, whereas including hereditary properties leads to a monotonic decay of the amplitude with time. The decay arises without any external damping and is caused by energy dissipation in the polymer matrix, described by the integral operator with kernel (9). The picture is the same for both types of support, differing only in the scale of the deflection. Hereditary properties do not affect the number of retained modes required: the upper and lower rows of Fig. 2 show the same behavior.

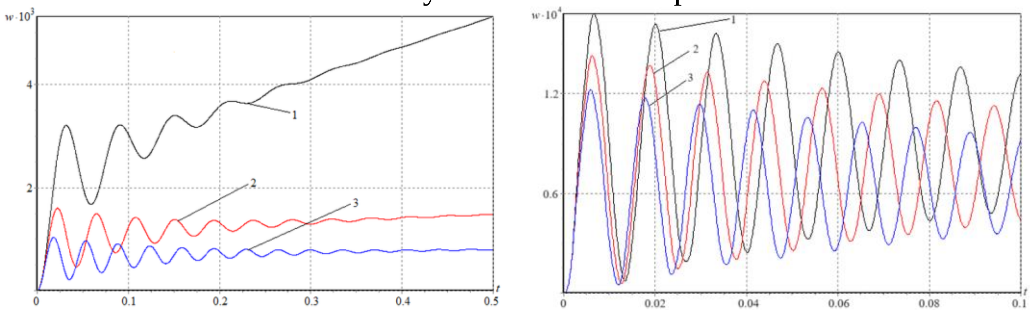
An increase in the aspect ratio $\lambda = a/b$ from 1 to 1.4 at constant thickness raises the amplitude of the vibrations and slows their decay. Both trends persist when the support is changed from simple to clamped, differing only in the scale of the deflection.



a) b)

Fig. 3. Deflection versus time for different values of the aspect ratio $\lambda = a/b$: a – simple support, b – clamping. Curves: 1 – $\lambda = 1$, 2 – $\lambda = 1.2$, 3 – $\lambda = 1.4$

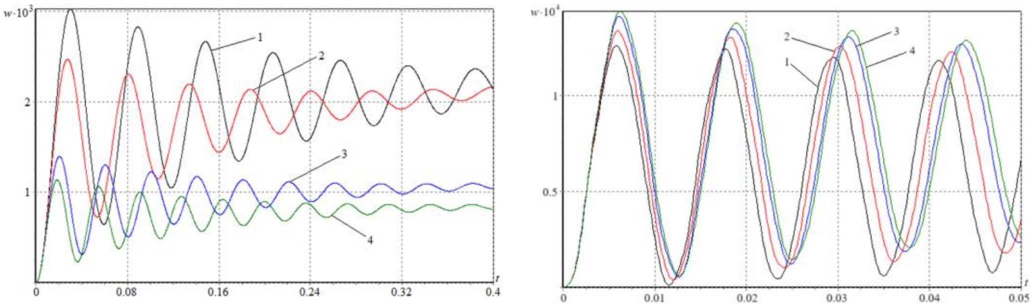
Reducing the thickness from 0.52 to 0.48 cm raises the vibration amplitude for both types of support. The result is expected: the bending stiffness is proportional to the cube of the thickness, and an 8% reduction in thickness noticeably increases the compliance of the structure.



a) b)

Fig. 4. Deflection versus time for different plate thicknesses: a – simple support, b – clamping. Curves: 1 – $h = 0.48$ cm, 2 – $h = 0.50$ cm, 3 – $h = 0.52$ cm

When the reinforcement angle is changed, the two types of support behave differently, and this is the main difference revealed in this work.

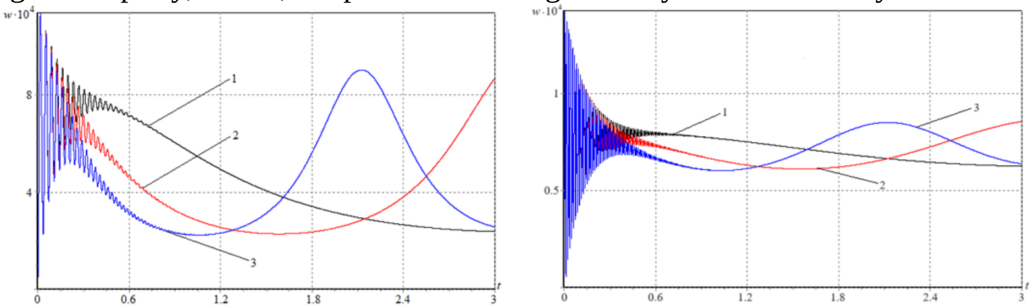


a) b)

Fig. 5. Effect of the reinforcement angle on the vibration process: a – simple support, b – clamping. Curves: 1 – $\theta = 0°$, 2 – $\theta = 15°$, 3 – $\theta = 30°$, 4 – $\theta = 45°$

For simple support, rotating the fibers from 0° to 45° reduces the deflection at the midpoint by roughly a factor of three: the amplitude of the first maximum falls from $3.2 \cdot 10^{-3}$ to $1.05 \cdot 10^{-3}$ m. For clamping, the same rotation changes the amplitude by about 10%, and in the opposite direction, while the main difference between the curves amounts to a phase shift.

Varying the frequency γ between 1 and 3 has almost no effect on the solution during the initial period of vibration: the curves do not diverge immediately. As the process develops, an increase in the frequency produces a phase shift to the left, which corresponds to a change in the vibration frequency of the plate. When the excitation frequency approaches the natural frequency, the amplitude grows rapidly, that is, the plate enters a region of dynamic instability.



a) b)

Fig. 6. Effect of the frequency of the external periodic load: a – simple support, b – clamping. Curves: 1 – $\gamma = 1$, 2 – $\gamma = 2$, 3 – $\gamma = 3$

4. Discussion

The main difference revealed by the calculations is the unequal sensitivity of the response to the reinforcement angle: for simple support, rotating the fibers from 0° to 45° reduces the deflection at the midpoint by roughly a factor of three, while for clamping it changes the amplitude by about 10% and in the opposite direction. The design consequence follows directly: choosing the reinforcement angle lowers the amplitude for simple support and ceases to work under clamping.

The difference in the number of retained modes required is explained by the form of the coordinate functions. The trigonometric functions used for simple support coincide with the natural vibration modes of a simply supported plate, so the first term of the series carries the main part of the solution. The clamped-beam functions contain hyperbolic terms concentrated near the edges, and the first eigenvalue of the beam function exceeds π by a factor of one and a half, so the clamped basis has a shorter wavelength. Both factors call for more series terms: nine modes compared with four. Hereditary properties do not affect this number, since the operator Γ^* changes the amplitude of the solution but not its spatial shape.

The decay obtained above is governed by the integral operator with kernel (9) and requires no external damping. For the adopted values $A = 0.0208$, $\alpha = 0.1$, and $\beta = 0.00166$ the total relaxation equals 0.375, that is, the long-term modulus is 0.625 times the instantaneous modulus; the amplitude decays fastest over the initial interval, where the weak singularity of the kernel gives the highest relaxation rate. The picture is the same for both types of support: the operator Γ^*

multiplies all components of Q_{ij}^* by the same factor and does not redistribute the stiffnesses between directions, so the effect of the reinforcement angle when hereditary properties are included is qualitatively the same as in the elastic problem.

Decay caused by energy dissipation in the material and requiring no external damping has been obtained previously for viscoelastic composite panels and anisotropic reinforced plates [16, 19], while the effect of hereditary properties on the dynamics of plates of variable thickness was noted in [17, 18]. The role of transverse shear and of the choice of the kinematic hypothesis in shaping the instability boundaries was shown in [11, 12], and the role of damping and support conditions in [13]. The present work differs in that the support conditions are treated not as one parameter among others, but as the subject of a direct comparison with all other conditions kept identical.

Methodologically, the work also differs from the line of work that goes back to reducing the problem to a Mathieu–Hill equation and then applying Bolotin's method [9, 10]: such a reduction assumes retention of a single mode and linearization, whereas the scheme adopted here works directly with the system of nonlinear integro-differential equations.

The limitations of the work are as follows. The comparison was made in terms of the dynamic response: critical loads were not determined. The calculations were performed for a single relaxation kernel and a single set of rheological parameters, and no experimental verification was carried out.

5. Conclusions

Comparing simple support and clamping within a single model leads to the following results.

- The sensitivity of the dynamic response to the reinforcement angle is governed by the support conditions. For a simply supported square plate, rotating the fibers from 0° to 45° reduces the deflection at the midpoint by roughly a factor of three, from $3.2 \cdot 10^{-3}$ to $1.05 \cdot 10^{-3}$ m. For clamping, the same rotation changes the amplitude by about 10%, and in the opposite direction, while the main difference between the curves amounts to a phase shift. Layup selection as a means of lowering the amplitude does not carry over from one support scheme to the other.
- Including hereditary properties turns the undamped vibrations of the elastic problem into a decaying process without external damping; for the adopted parameters the total relaxation is 0.375. The picture is the same for both types of support, and hereditary properties do not affect the number of modes required.
- For the clamped plate, convergence is reached with nine retained modes compared with four for simple support. The reason lies in the structure of the beam functions: they contain hyperbolic terms concentrated near the edges, and the first eigenvalue of the beam function exceeds π by about a factor of one and a half.
- The effect of the geometric parameters is qualitatively the same for both types of support: raising the aspect ratio from 1 to 1.4 increases the amplitude and slows the decay; reducing the thickness from 0.52 to 0.48 cm increases the amplitude; and raising the frequency of the external load produces a phase shift.

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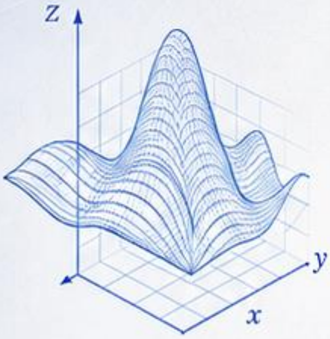


$$\frac{dx}{dt} = Ax + b(t)$$

$$\frac{\partial u}{\partial t} = \alpha \nabla^2 u + f$$

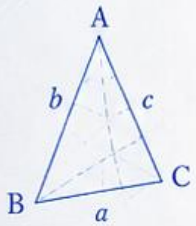
$$\sum_{i=1}^{n} a_i x_i + b$$

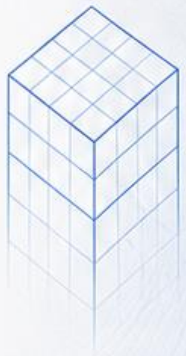
$$f(x) = \int_a^b f(x)\, dx$$

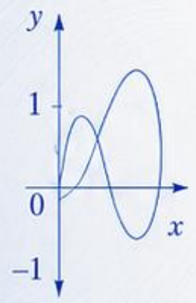
CONTENTS

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

$$\sum_{i=1}^{n} w_i x_i$$







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