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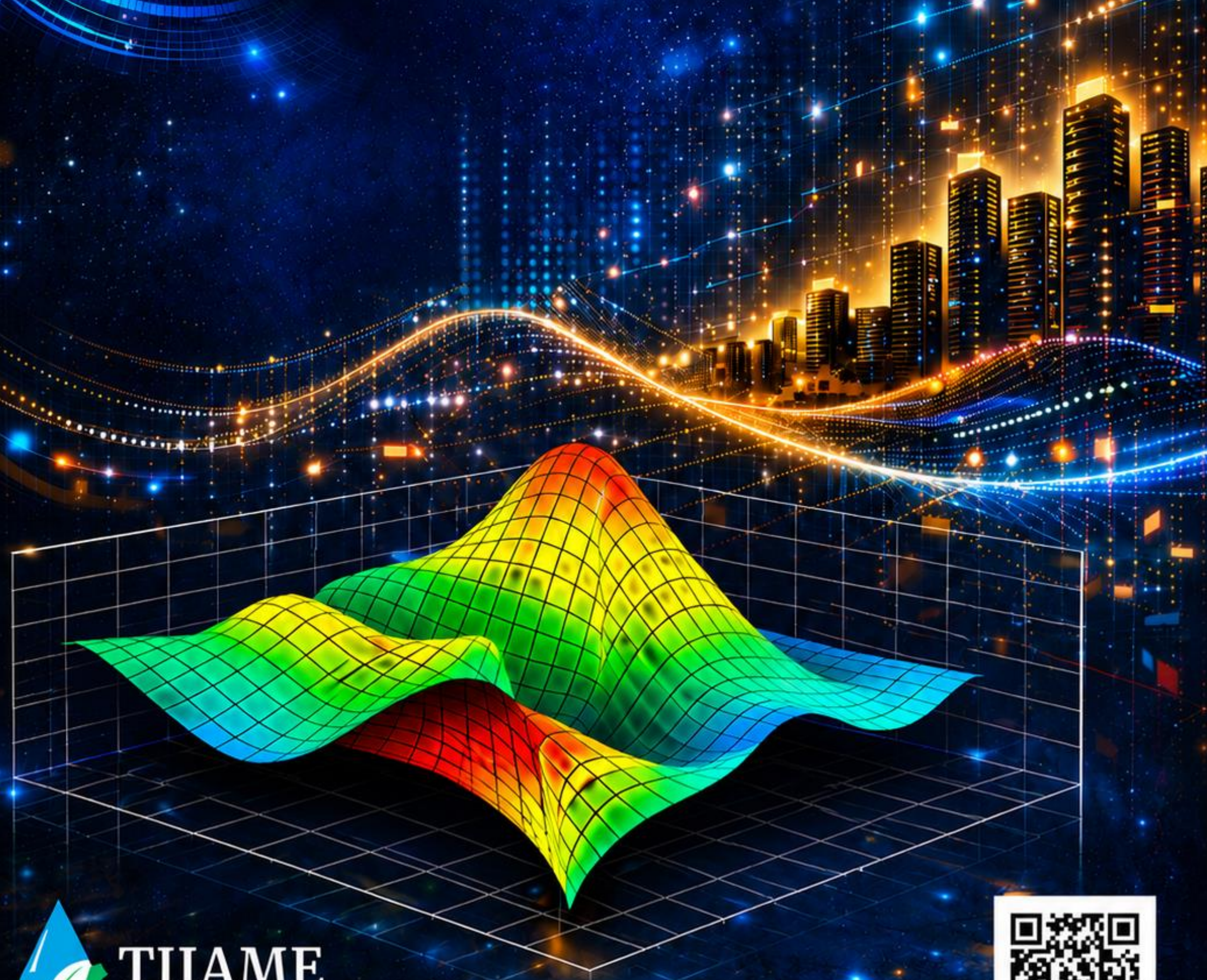

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Assessment of the strength of a cylindrical shell under the influence of internal viscous fluid

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ABSTRACT

In this article, the stress–strain state and strength of a transversally isotropic cylindrical shell interacting with an internal viscous fluid are investigated. Based on a mathematical model describing the non-stationary interaction between the shell and the fluid, the components of the stress tensor have been expressed analytically, and an algorithm for their numerical calculation has been developed. The laws governing the distribution of radial and torsional stresses along the shell length under various torque values and under external radial and longitudinally radial forces have been analyzed. Based on the obtained results, the equivalent stress was determined according to the Von Mises criterion, and the shell's strength was evaluated. The calculation results show that the internal viscous fluid has a significant effect on the distribution of stresses and that the structure under consideration satisfies the strength requirements under the given loading conditions..

1. Introduction

Cylindrical shells made from transversally isotropic materials are one of the structural elements widely used in mechanical engineering, aviation, energy and the oil and gas industries. The operational reliability of such structures depends largely on accurately assessing their state of stress - strain under loading. In particular, in cases where a viscous fluid is present within the shell, the interaction between the fluid and the elastic structure gives rise to complex hydro elastic phenomena, which significantly affect the stress distribution and strength characteristics. Therefore, the investigation of the stress state and the assessment of the strength of transversally isotropic cylindrical shells interacting with an internal viscous fluid is a pressing scientific and practical problem.

In the scientific research conducted in this field, particular attention has been paid to the static and dynamic behavior of cylindrical shells, the interaction between the fluid and the structure, and the improvement of strength criteria. One of the first studies conducted in this field [1] is cited in the work, in which a mathematical model describing the propagation processes of waves in transversely isotropic cylindrical structures was developed and the problem was analyzed using theoretical and numerical methods. The obtained results, in addition to developing the theoretical foundations for assessing the strength of cylindrical structures, provide an important scientific basis for further research into the mechanical state of cylindrical shells under the action of an internal

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viscous fluid. [2] a numerical calculation method that takes into account geometric nonlinearity has been developed to analyze the elastoplastic deformed state of laminated circular shells made from transversally isotropic materials. The approach proposed by the authors contributes to the development of research on determining the mechanical state and assessing the strength of cylindrical shells under complex loads, including serving as an important theoretical basis for solving problems that take into account the effect of the internal viscous fluid. [3, 4] investigated the interaction between a viscous fluid and an elastic layer based on three-dimensional linearized Navier-Stokes equations and classical elastic theory, the effect of fluid viscosity on the propagation characteristics of waves was evaluated using numerical methods. The results obtained confirm that viscous fluid has a significant effect on the dynamic characteristics of elastic structures; it provides an important theoretical basis for research into the analysis of the mechanical state and the assessment of the strength of cylindrical shells under the influence of internal viscous fluid. [5] a refined theory has been developed in the work for the axial and transverse vibrations of cylindrical shells, taking into account the effects of rotational inertia and transverse displacement deformations. Also included are algorithms for determining the stress-strain state in an arbitrary cross-section of the shell, as well as the laws of vibration and wave propagation, they constitute an important theoretical foundation for research into the strength assessment of cylindrical shells under the action of an internal viscous fluid.

The analysis of the above research indicates that, the development and refinement of refined mathematical models that account for the fluid–structure interaction when assessing the strength of cylindrical shells interacting with internal viscous fluid remains a pressing scientific challenge.

2. Mathematical model

In the next stage of the study, with the aim of analytically expressing the stress state, we express the components of the stress tensor at the points of the transversally isotropic cylindrical shell interacting with the internal viscous fluid through the sought-after functions. For a transversally isotropic medium, the stress tensor components are written as follows [5].

$$\begin{gathered}\sigma_{rr}=C_{11}\frac{\partial U_r}{\partial r}+C_{12}\frac{U_r}{r}+C_{13}\frac{\partial U_z}{\partial z};\ \sigma_{\theta\theta}=C_{12}\frac{\partial U_r}{\partial r}+C_{11}\frac{U_r}{r}+C_{13}\frac{\partial U_z}{\partial z};\\ \sigma_{zz}=C_{13}\left(\frac{\partial U_r}{\partial r}+\frac{U_r}{r}\right)+C_{33}\frac{\partial U_z}{\partial z};\ \sigma_{rz}=C_{44}\left(\frac{\partial U_r}{\partial z}+\frac{\partial U_z}{\partial r}\right).\end{gathered}\tag{1}$$

In this $\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}, \sigma_{rz}$ are the radial, torsional, longitudinal and longitudinal-radial components of the stress tensor, respectively, $C_{11}, C_{12}, C_{13}, C_{33}, C_{44}$ are Elastic modulus for a transversally isotropic body, U_r, U_z are the radial and longitudinal components of the displacement vector, r is radius of the shell's mean surface.

After substituting the displacement functions into the expressions for the components of the stress tensor and performing the necessary mathematical simplifications, the stress components take the following form.

$$\left[\rho B_1 C_{11}^{-1}\frac{\partial^2}{\partial t^2}-C_{44}C_{11}^{-1}B_1\frac{\partial^2}{\partial z^2}\right]\sigma_{rr}=\left[\bar{A}_{11}\frac{\partial U_{z,0}}{\partial z}+\bar{B}_{11}U_{r,0}+\bar{N}_{11}\frac{\partial U_{z,1}}{\partial z}+\bar{M}_{11}U_{r,1}\right];$$

$$\left[\rho B_1 C_{11}^{-1}\frac{\partial^2}{\partial t^2}-C_{44}C_{11}^{-1}B_1\frac{\partial^2}{\partial z^2}\right]\sigma_{\theta\theta}=\left[\bar{A}_{21}\frac{\partial U_{z,0}}{\partial z}+\bar{B}_{21}U_{r,0}+\bar{N}_{21}\frac{\partial U_{z,1}}{\partial z}+\bar{M}_{21}U_{r,1}\right];$$

$$\left[\rho B_1 C_{11}^{-1}\frac{\partial^2}{\partial t^2}-C_{44}C_{11}^{-1}B_1\frac{\partial^2}{\partial z^2}\right]\sigma_{zz}=\left[\bar{A}_{31}\frac{\partial U_{z,0}}{\partial z}+\bar{B}_{31}U_{r,0}+\bar{N}_{31}\frac{\partial U_{z,1}}{\partial z}+\bar{M}_{31}U_{r,1}\right];$$

$$\left[\rho C_{11}^{-1}B_1\frac{\partial^3}{\partial z\partial t^2}-B_1 C_{44}C_{11}^{-1}\frac{\partial^3}{\partial z^3}\right]\sigma_{rz}=\left[\bar{A}_{41}\frac{\partial U_{z,0}}{\partial z}+\bar{B}_{41}U_{r,0}+\bar{N}_{41}\frac{\partial U_{z,1}}{\partial z}+\bar{M}_{41}U_{r,1}\right];$$

(2)

here, $B_1=\left(C_{13}+C_{44}\right)/C_{11}$. $\bar{A}_{i1},\ \bar{B}_{i1},\ \bar{N}_{i1},\ \bar{M}_{i1}\ \ (i=1,2,3,4)$ are differential operators depending on higher-order derivatives with respect to the time t and the coordinate z, i.e.

$$\bar{A}_{11}=a_{11}^{'}\frac{\partial^4}{\partial t^4}+a_{12}^{'}\frac{\partial^4}{\partial t^2\partial z^2}+a_{13}^{'}\frac{\partial^4}{\partial z^4}+a_{14}^{'}\frac{\partial^2}{\partial t^2}+a_{15}^{'}\frac{\partial^2}{\partial z^2},....,$$

$$\bar{B}_{11}=b_{11}^{'}\frac{\partial^4}{\partial t^4}+b_{12}^{'}\frac{\partial^4}{\partial t^2\partial z^2}+b_{13}^{'}\frac{\partial^4}{\partial z^4}+b_{14}^{'}\frac{\partial^2}{\partial t^2}+b_{15}^{'}\frac{\partial^2}{\partial z^2},....,$$

$$\bar{N}_{11}=n_{11}^{'}\frac{\partial^4}{\partial t^4}+n_{12}^{'}\frac{\partial^4}{\partial t^2\partial z^2}+n_{13}^{'}\frac{\partial^4}{\partial z^4}+n_{14}^{'}\frac{\partial^2}{\partial t^2}+n_{15}^{'}\frac{\partial^2}{\partial z^2},...,$$

$$\bar{M}_{11}=m_{11}^{'}\frac{\partial^4}{\partial t^4}+m_{12}^{'}\frac{\partial^4}{\partial t^2\partial z^2}+m_{13}^{'}\frac{\partial^4}{\partial z^4}+m_{14}^{'}\frac{\partial^2}{\partial t^2}+m_{15}^{'}\frac{\partial^2}{\partial z^2},...$$

$a_{ij}^{'},b_{ij}^{'},n_{ij}^{'}$ and $m_{ij}^{'}$ coefficients the geometric dimensions of the cylindrical shell, its physical-mechanical properties, as well as parameters dependent on the density of the viscous fluid within the shell and its dynamic and kinematic viscosity properties. These coefficients are determined on the basis of the following expressions:

$$a_{11}^{'}=B_1\left(B_1C_{11}-C_{13}+\frac{C_{12}}{4}\right)\frac{1}{C_{11}C_{44}}\frac{r^2}{4},...,$$

$$m_{11}^{'}=B_1\xi\left(\eta_{2,0}(r)C_{11}+\eta_{1,0}(r)\frac{C_{12}}{2}\right)\frac{1}{C_{11}^2},....$$

$$b_{11}^{'}=\frac{1}{C_{11}C_{44}}\left(B_1C_{11}+C_{13}-\frac{B_1C_{12}}{4}\right)\frac{r^2}{4}+\frac{1}{C_{11}}\left(\frac{1}{C_{11}}+\frac{1}{C_{44}}\right)\left(B_1C_{11}+\frac{C_{12}}{4}B_1-C_{13}\right)\frac{r^2}{4}+C_{13}\frac{r^2}{4}\frac{1}{C_{11}^2},...,$$

$$n_{11}^{'}=\xi\left[\frac{B_1}{C_{11}}\left(\frac{1}{C_{11}}+\frac{1}{C_{44}}\right)\left(\eta_{2,1}(r)\left(B_1C_{11}-C_{13}\right)+\eta_{1,1}(r)\frac{B_1C_{12}}{4}\right)\frac{r^2}{4}+\eta_{2,1}(r)B_1\frac{C_{13}}{C_{11}^2}\frac{r^2}{4}\right],...,$$

Using the equations in (2), it is possible to determine the stresses at any point on a transversally isotropic cylindrical shell during longitudinal-radial vibrations, in interaction with an internal viscous fluid.

To investigate the shell’s stress state using numerical calculation methods, the stress components are expressed as finite difference equations in terms of the displacement and deformation parameters. This approach makes it possible to analyze in detail the mechanical interaction processes that occur between the cylindrical shell and the viscous fluid within it.

3. Results and discussion

Using the derived solutions of the longitudinal-radial vibration equation [6] for the system under consideration, together with the initial and boundary conditions formulated for the problem [7], we produce graphs of the stress distribution $\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}$ and σ_{rz}. Initially, the torque acting on the three free ends of the cylindrical shell made of aluminium material at values of $5 \cdot 10^3 N \cdot m$, $10 \cdot 10^3 N \cdot m$, $15 \cdot 10^3 N \cdot m$, as well as taking into account the f_r radial forces and the f_{rz} longitudinal-radial surface forces acting along the surface graphs of the $\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}$ and σ_{rz} stresses versus the z coordinate are constructed, and their distribution characteristics are analysed.

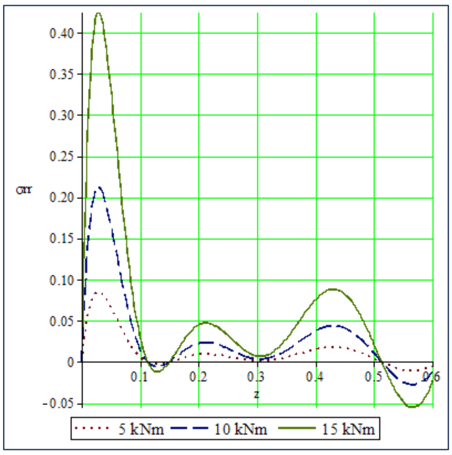

Fig. 1. Characteristic variation of the stress tensor component along the z-axis under various torque values σ_{rr}.

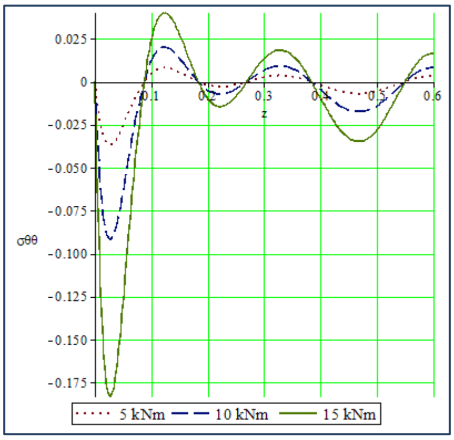

Fig. 2. Characteristic variation of the stress tensor component along the z-axis under various torque values $\sigma_{\theta\theta}$.

Fig. 1 shows a graph of the coordinate variation of the stress tensor component for a circular cylindrical shell made of aluminium. As the graphs show, the maximum value of the radial stress σ_{rr}, under all three moments and the external radial and axial-radial forces, is observed in the initial section at $z \approx 0$, i.e. along the shell length. Subsequently, as the z-coordinate increases, this stress magnitude gradually decreases. In the subsequent intervals of the z-coordinate, the radial stress σ_{rr} value gradually decreases and at some points even becomes negative. This indicates that compressive and tensile deformations are alternating in the radial direction of the shell. Furthermore, it is observed that as the applied torque increases, the maximum value of σ_{rr} also increases accordingly: It is highest at $M = 15\, kN \cdot m$ and lowest at $M = 5 kN \cdot m$. This relationship shows that in the elastic deformation region of the material there is an almost linear relationship between torque and stress. Such results are of great importance for assessing the strength of the structure and potential crack zones. Overall, the analysis results σ_{rr} scientifically substantiate the uneven distribution of radial stress z along the coordinate axis and the significant variation in stress states across different cross-sections of the element.

Fig. 2 shows the variation of the torsional stress component z with the $\sigma_{\theta\theta}$ coordinate for the longitudinal-radial vibrations of a transversely isotropic cylindrical shell in a non-stationary interaction with an internal viscous fluid. As the graph shows, the gradual reduction in the amplitude of the stress oscillations along the z-coordinate is explained by the dissipative properties of the internal viscous fluid. This case confirms that the applied boundary condition is consistent with the physical context and that the effect of external forces diminishes with increasing distance. From an engineering standpoint, the most critical area of the structure is the boundary where loads combine to exert their maximum effect, and it is precisely at this point that the likelihood of fatigue cracks developing is highest. According to the results obtained, even though a torque of $15\, kN \cdot m$

represents a high operating regime, the calculated stresses do not exceed the permissible limit and the structure retains an adequate safety factor. In general, for the case of a non-stationary interaction between an internal curved fluid and a transversally isotropic cylindrical shell, it describes the complex hydroelastic stress state arising under torque and external radial and longitudinally radial forces.

The analysis of the stress and strain state, and the assessment of the strength of cylindrical shells made from transversally isotropic materials, is one of the most pressing scientific and practical problems in modern mechanics and structural theory. One of the most widely used criteria for determining the equivalent stress to assess the load-bearing capacity of such structures is the von Mises strength criterion [8]. In the cylindrical coordinate system, this equivalent stress is determined by the following mathematical expression:

$$\sigma_{eq}=\sqrt{\frac{1}{2}\left[\left(\sigma_{rr}-\sigma_{\theta\theta}\right)^2+\left(\sigma_{\theta\theta}-\sigma_{zz}\right)^2+\left(\sigma_{zz}-\sigma_{rr}\right)^2\right]+3\sigma_{rz}^2}; \qquad (3)$$

In this σ_{eq} is equivalent stress. Analysis of the graphs in Figures 1 and 2 shows that the maximum values of the stresses generated under torque and external forces are determined as follows, i.e. $\sigma_{rr}^{\max}=0,42;\ \sigma_{\theta\theta}^{\max}=0,03;\ \sigma_{zz}^{\max}=0,065;\ \sigma_{rz}^{\max}=0,004$. When the maximum stress values determined from the graphs are substituted into equation (3), the equivalent stress, according to the calculation results, is approximately $\sigma_{eq}=0,37$.

Now the shell's strength is analyzed for various torque values. To this end, the stress values determined from the graphical results presented in Figures 1 and 2 are examined. According to the graph analysis, the maximum radial stress $\sigma_{rr}=0,65$, the absolute maximum torsional stress $\sigma_{\theta\theta}=0,25$, the maximum longitudinal stress $\sigma_{zz}=0,11$ and the absolute maximum longitudinal-radial stress $\sigma_{rz}=0,006$ are observed. The stability condition of the structure is expressed by the following inequality: $\sigma_{eq}\le\left[\sigma\right]$, here, $\left[\sigma\right]$ is denotes the allowable stress for the material. For all the above cases, when the strength condition is checked, it is found that the equivalent stress values determined from the calculations are less than the allowable stresses. These results indicate that the cylindrical shell under consideration fully satisfies the strength requirements under the given load and ensures its operational reliability.

The results of the theoretical and computational analyses indicate that the cylindrical shell made of transversally isotropic material satisfies the strength requirements under the given loading and ensures its reliable operation. The obtained results are of significant scientific and practical importance for assessing the stress state of transversally isotropic shell structures, substantiating their strength and further improving engineering calculation methods.

4. Conclusion

In this research, based on a mathematical model describing the stress state of a transversally isotropic cylindrical shell interacting with an internal viscous fluid, the components of the stress tensor were determined and the laws governing their distribution along the shell length were analyzed. The computational results show that as the torsional moment and external loads increase, the maximum stresses also rise, and the highest stresses occur in the boundary regions where the loads are applied. The assessment, conducted using the Von Mises strength criterion, determined that the calculated equivalent stresses do not exceed the permissible values, confirming that the cylindrical shell under consideration has an adequate safety factor under the given operating loads. The results obtained are of theoretical and practical importance for improving the engineering calculations for the design and reliability assessment of cylindrical shells subjected to internal pressurised fluid.

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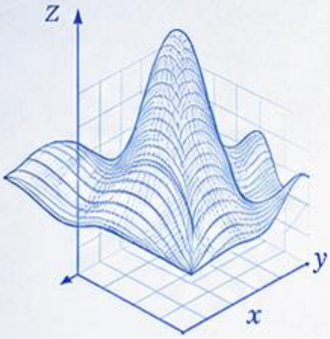


$$\frac{dx}{dt} = Ax + b(t)$$

$$\frac{\partial u}{\partial t} = \alpha \nabla^2 u + f$$

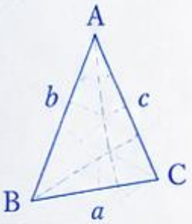
$$\sum_{i=1}^{n} a_i x_i + b$$

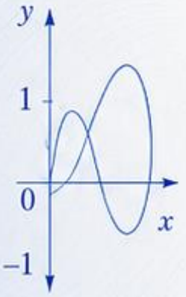
$$f(x) = \int_a^b f(x)\, dx$$

CONTENTS

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

$$\sum_{i=1}^{n} w_i x_i$$





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