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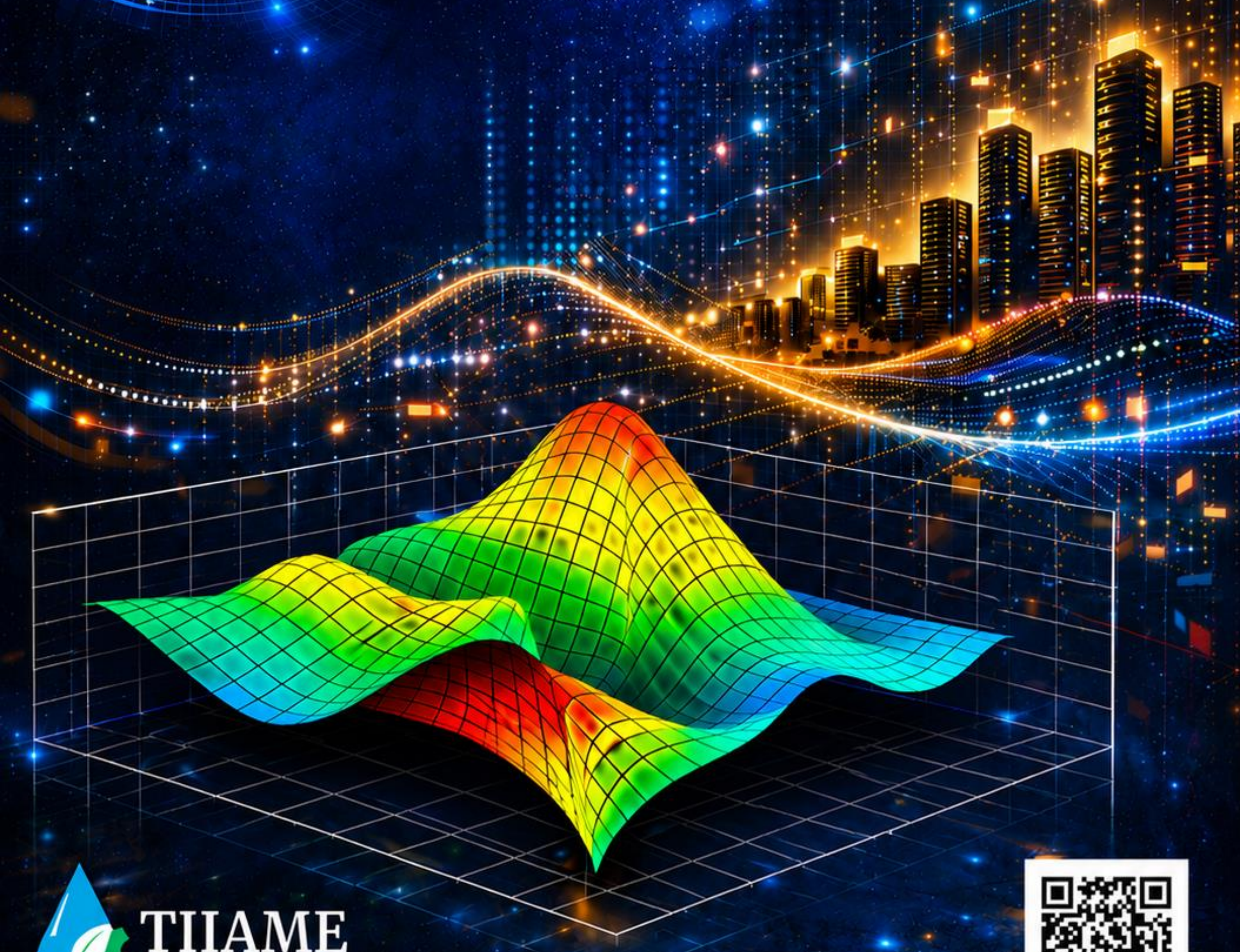

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Comparison of the exact and asymptotic solutions to the problem of oscillatory flow of a viscous fluid in a cylindrical pipe

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ABSTRACT

The oscillatory flow of an incompressible viscous fluid in a cylindrical pipe under a given harmonic oscillation of the average velocity across the cross section was studied. The transfer function and amplitude-phase frequency characteristics were determined. Using these functions, the effect of the oscillation frequency on the shear strength relative to the average velocity across the cross section was evaluated. By comparing the exact and asymptotic solutions, new asymptotic formulas were proposed, which are of great importance for studying the oscillatory flow of viscoelastic fluids in cylindrical pipes. The obtained results can be used in biomedical research for the estimation of pressure losses, in the analysis of pulsatile flows in oil and gas pipelines for the optimization of pump parameters, as well as in the medical field for modeling the pulsatile flow in the assessment of the elasticity of human blood vessels.

1. Introduction

Pneumatic micropumps used in medical and biological devices are often used to study the oscillatory flow of viscous fluids in a cylindrical tube under the influence of harmonic oscillations of the fluid flow [1,2]. These systems are designed for diagnosing the functioning of various human organs, as well as for the targeted delivery of drugs. In such systems, it can be cost-effective to set up a pulsating flow rate. Currently, there is no information on the effect of flow pulsation on changes in hydraulic resistance and frictional drag coefficients [3-5]. However, this research is very important for calculating pressure gradients and other hydrodynamic properties, as well as for biomedical and other technological research.

In [6], unsteady pulsating flows under the influence of harmonic pressure changes in an infinite circular cylindrical pipe were studied. Formulas for the distribution of the volume and velocity of the fluid were obtained. Pulsating flows filled with incompressible fluid in a rectangular channel were studied in [7]. Optimal differences in the scheme were determined and data on the amplitude and phase of the velocity oscillations were obtained.

The work [8] examines the fundamentals of numerical modeling of isothermal and non-isothermal flows of non-Newtonian fluids with a free surface, which consists of formulating

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mathematical models and developing methods for calculating fluid flows when filling flat and axisymmetric channels.

In [9], the motion of a viscoelastic fluid along a long pipe under the action of an oscillatory pressure gradient was investigated. Distinctive features of this motion were demonstrated in comparison with the corresponding motion of a Newtonian fluid. Oscillatory flows of Maxwell and Oldroyd-B viscoelastic fluids were studied in [10]. Many interesting properties were discovered that are not present in the flows of Newtonian fluids.

The problem of unsteady oscillatory flow of a viscoelastic fluid in a flat channel based on the Maxwell model is considered in [11-17]. Formulas for determining the dynamic and frequency characteristics are obtained. It is shown that the viscoelastic properties of the fluid, as well as its acceleration, are limiting factors for the use of a quasi-steady approach. Electrokinetic flow of viscoelastic fluids in a flat channel under the action of an oscillating pressure gradient was studied in [18]. It was established that during oscillating flow of a viscoelastic fluid, a resonance phenomenon occurs, when the elastic properties of the Maxwell fluid predominate.

The studies listed above primarily examine the fluid velocity field using various methods of changing the pressure gradient. During the oscillatory motion of a viscous fluid, the changes in shear stresses that arise when harmonic oscillations are superimposed on the flow have been relatively little studied.

In this regard, this paper examines the oscillatory flow of a viscous fluid in a cylindrical pipe for a given law of average velocity variation across the pipe cross-section. The transfer function of the amplitude-phase frequency response (APFR) is determined. Using this function, the dependence of the transient shear stress on the wall on the dimensionless oscillation frequency and acceleration of the fluid is studied. By comparing the exact solution to the asymptotic solution, new asymptotic formulas are proposed that are of great importance in studying the oscillatory flow of viscoelastic fluid in a cylindrical pipe.

2. Problem Statement and Solution Method

We formulate the problem of slow oscillatory flow of a viscous fluid in a cylindrical pipe of infinite length. Let us denote the radius of the pipe by . The axis runs horizontally in the middle of the pipe along the flow, and the axis is directed perpendicular to the axis. The differential equation for the motion of a viscous incompressible fluid has the following form [19-21]

$$\rho \frac{\partial \vartheta_x}{\partial t} = -\frac{\partial p}{\partial x} + \eta \left(\frac{\partial^2 \vartheta_x}{\partial r^2} + \frac{1}{r}\frac{\partial \vartheta_x}{\partial r}\right) \tag{1}$$

where ϑ_x- are the longitudinal velocity; p -pressure; ρ -density; η -dynamic viscosity; t -time.

We assume that the oscillatory flow of a viscous fluid occurs due to the longitudinal velocity averaged over the cross-section of the pipe.

$$< \vartheta_x >= a_{\vartheta_x} cos\omega t = Re\, a_{\vartheta_x} e^{i\omega t} \tag{2}$$

where a_{ϑ_x}- are the amplitudes of the longitudinal velocity averaged over the pipe cross-section; ω - is the oscillation frequency. The no-slip condition is satisfied at the pipe walls, and the longitudinal velocity is limited at the center. Then the boundary conditions are:

$$\begin{aligned} &\vartheta_x = 0 \quad \text{at} \quad r = R \\ &\vartheta_x < \infty \quad \text{at} \quad r = 0 \end{aligned} \tag{3}$$

Due to the linearity of equation (1) of longitudinal velocity, pressure, and shear stress on the wall can be written as follows

$$\vartheta_x(r,t) = Re\, \vartheta_{x1}(r) e^{i\omega t}, p(x,t) = Re\, p_1(x) e^{i\omega t}, \tau(t) = Re\, \tau_1\, e^{i\omega t} \tag{4}$$

Substituting these expressions into equation (1), we obtain

$$\left(\frac{\partial^2 \vartheta_{x1}(r)}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right) - \frac{\rho i \omega}{\eta} \vartheta_{x1}(r) = \frac{1}{\eta}\frac{\partial p_1(x)}{\partial x}, \tag{5}$$

$\alpha_0 = \sqrt{\frac{\omega}{\nu}} R$ - Womersley oscillation number (dimensionless oscillation frequency); ν - kinematic viscosity.

Solving equation (5) taking into account the boundary conditions (3), we have

$$\vartheta_{x_1}(r) = \frac{1}{\rho i\omega}\left(-\frac{\partial p_1(x)}{\partial x}\right)\left(1 - \frac{J_0\left(i^{\frac{3}{2}}\alpha_0)\frac{r}{R}\right)}{J_0\left(i^{\frac{3}{2}}\alpha_0)\right)}\right) \tag{6}$$

Using equation

$$\tau_1 = -\eta\frac{\partial\vartheta_{x_1}(r)}{\partial r}|r = R \tag{7}$$

we find the shear stress on the wall

$$\tau_1 = -R\left(-\frac{\partial P}{\partial x}\right)\frac{1}{i\alpha_0^2}\left(\frac{i^{\frac{3}{2}}\alpha_0 J_1\left(i^{\frac{3}{2}}\alpha_0\right)}{J_0\left(i^{\frac{3}{2}}\alpha_0\right)}\right) \tag{8}$$

By multiplying both parts of equation (6) $2\pi r$ and integrating them from zero to R, over the variable r, we find formulas for the liquid flow rate

$$Q_1 = \pi R^2\left[\frac{1}{\rho i\omega}\left(-\frac{\partial p_1(x)}{\partial x}\right)\left(1 - \frac{2J_1\left(i^{\frac{3}{2}}\alpha_0\right)}{\left(i^{\frac{3}{2}}\alpha_0\right)J_0\left(i^{\frac{3}{2}}\alpha_0\right)}\right)\right] \tag{9}$$

and taking into account formula (9) $Q_1 = \pi R^2 < \vartheta_{x_1} >$, we find the longitudinal velocity averaged over the cross-section of the pipe

$$< \vartheta_{x1} >= \frac{1}{\rho i\omega}\left(-\frac{\partial p_1(x)}{\partial x}\right)\left(1 - \frac{2J_1\left(i^{\frac{3}{2}}\alpha_0\right)}{\left(i^{\frac{3}{2}}\alpha_0\right)J_0\left(i^{\frac{3}{2}}\alpha_0\right)}\right) \tag{10}$$

Here $\rho i\omega$ we can write it as

$$\rho i\omega = i\frac{\omega}{\nu}R^2 \cdot \frac{\eta}{R^2} = i\alpha_0^2\frac{\eta}{R^2}$$

Then formula (10) takes the form:

$$< \vartheta_{x1} >= \frac{R}{4\eta}\tau_1 \cdot \frac{4J_2(i^{\frac{3}{2}}\alpha_0)}{\left(i^{\frac{3}{2}}\alpha_0\right)J_1(i^{\frac{3}{2}}\alpha_0)} \tag{11}$$

We define the transfer function $W_{\tau_1,\vartheta_{x1}}(i\omega)$ using formulas (11), as

$$W_{\tau_1,\vartheta_{x_1}}(i\omega) = \frac{R}{4\eta}\frac{\tau_1(i\omega)}{<\vartheta_{x_1}(i\omega)>} \tag{12}$$

From equation (11) we obtain

$$W_{\tau_1,\vartheta_{x_1}}(i\omega) = \frac{R}{4\eta}\frac{\tau_1(i\omega)}{<\vartheta_{x_1}>} = \frac{\left(i^{\frac{3}{2}}\alpha_0\right)J_1\left(i^{\frac{3}{2}}\alpha_0\right)}{4J_2(i^{\frac{3}{2}}\alpha_0)} \tag{13}$$

The transfer function (13) is called the amplitude-phase frequency response (APFR). These functions allow us to determine the time dependence of the shear stress on the pipe wall for a given law of change in the longitudinal velocity averaged over the flow cross-section. As is well known, in most cases, when solving transient problems, shear stresses on the wall obtained under quasi-steady fluid flow conditions are used. In real cases, such assumptions are valid when the distribution of local velocities over the flow cross-section has a parabolic distribution law. In this case, the shear stress on the pipe wall oscillates in phase with the oscillation of the averaged longitudinal velocity over the flow cross-section.

Then the value $\tau_{o,kc}$ can be calculated using the formula

$$\tau_{o,kc} = \frac{4\eta}{R} < \vartheta_{x1} > \tag{14}$$

And instead of the quasi-stationary flow of tangential shear stress on the wall$\tau_{o,kc}$,, we can accept

$$\tau_{o,kc} = \tau_{\text{НС}} \tag{15}$$

Thus, relation (15) makes it possible to replace the quantity $\tau_{\text{НС}}$with the value $\tau_{o,kc}$, only under the condition that the actual distribution of local velocities over the flow cross-section differs little from the quasi-stationary one. However, in many cases, the local velocity distribution law in an unsteady flow differs significantly from the quasi-steady state. Most studies [8-15, 18-21] have shown that, in oscillatory laminar flow in a cylindrical pipe, the change in local velocities in the near-dock layers is faster than the change in local velocities in the central layers. Due to the change in the law of distribution of local velocities across the channel cross-section, the oscillatory flow value $\tau_{нс}$ actually differs significantly from $\tau_{0кс}$. The most complete understanding of the dependence of $\tau_{нс}$ on $< \vartheta_{x1} >$ the linear model of a non-stationary flow can be obtained using the transfer function (13).

3. Calculation results and analysis.

Using the transfer function (13), we investigate the dependence of the shear stress on the longitudinal velocity averaged over the cross-section of the pipe, when the law of change in longitudinal velocity is given in the form

$$< \vartheta_{x1} >= a_{\vartheta_{x1}} \cos \omega t \qquad (16)$$

where $a_{\vartheta_{x1}}$- is the amplitude of the longitudinal velocity averaged over the cross-section of the pipe. Due to the linearity of equations (16), the change in the shear stress will also be harmonic, but in the general case, shifted in phase with respect to $< \vartheta_{x1} >$. Then, the change in shear stress is determined as follows

$$\tau_1 = a_{\tau_1} \cos(\omega t + \phi_{\tau_1}) \qquad (17)$$

where a_{τ_1}- is the amplitude of the shear stress; ϕ_{τ_1}- phase shift between the quantities τ_1 and $< \vartheta_{x1} >$

Using the ratio

$$\cos(\omega t + \phi_{\tau_1}) = \cos \omega t \cos \phi_{\tau_1} - \sin \omega t \sin \phi_{\tau_1}$$

and considering that

$$\frac{\partial < \vartheta_{x1} >}{\partial t} = -a_{\vartheta_{x1}} \omega \sin \omega t$$

Equation (19) will take the form

$$\tau_1 = (\frac{a_{\tau_1}}{a_{\vartheta_{x1}}} \cos \phi_{\tau_1}) < \vartheta_{x1} > +(\frac{a_{\tau_1}}{a_{\vartheta_{x1}}} \sin \phi_{\tau_1}) \frac{1}{\omega} \frac{\partial < \vartheta_{x1} >}{\partial t} \qquad (18)$$

The quantities $(\frac{a_{\tau_1}}{a_{\vartheta_{x1}}} \cos \phi_{\tau_1})$ and $\frac{a_{\tau_1}}{a_{\vartheta_{x1}}} \sin \phi_{\tau_1}$ are the real and imaginary parts of the transfer function (13), respectively, therefore from (16) we obtain

$$W_{\tau_1, \vartheta_{x_1}} = \frac{1}{4} (\frac{\left(i^{\frac{3}{2}} \alpha_0\right) J_1 \left(i^{\frac{3}{2}} \alpha_0\right)}{J_2 (i^{\frac{3}{2}} \alpha_0)}) = \chi + \beta i \qquad (19)$$

Here $\chi = (\frac{a_{\tau_1}}{a_{\vartheta_{x1}}} \cos \phi_{\tau_1})$ - are the real and $\beta = \frac{a_{\tau_1}}{a_{\vartheta_{x1}}} \sin \phi_{\tau_1}$- imaginary parts of the transfer function (13).

Taking into account (19) in (18), we obtain

$$\frac{R}{4\eta} \frac{\tau_1}{< \vartheta_{x1} >} = W_{\tau_1, \vartheta_{x1}} = \chi + \beta \frac{1}{\omega} K_н \qquad (20)$$

Here - $K_н = \frac{\partial < \vartheta_{x1} >}{< \vartheta_{x1} > \partial t}$ - parameter determines the relative acceleration of the liquid, χ and β- are dimensionless quantities, t - dimensional quantities, so it needs to be converted to a dimensionless form using the transformation

$$t = \frac{R^2 \rho}{8\eta} t^* \qquad (21)$$

Taking into account (15) and (21) on (20), we obtain the calculation formula

$$\frac{\tau_{нс}}{\tau_{0кс}} = \chi + \frac{8\beta}{\alpha_0^2} K^*{}_н \qquad (22)$$

Here $K^*{}_н = \frac{\partial < \vartheta_{x1} >}{< \vartheta_{x1} > \partial t^*}$, $\tau_{0кс} = \frac{4\eta}{R} < \vartheta_{x1} >$ and $\tau_1 = \tau_{нс}$.

χ and β determined using formula (19).

For calculations, instead of Bessel functions, one can use Thomson functions, taking into account that

$$J_n(i^{3/2}\alpha_0) = ber_n(\alpha_0) + ibei_n(\alpha_0) \tag{23}$$

then the amplitude-phase frequency characteristic (19) will take the form

$$W_{\tau_1,\vartheta_{x_1}}(i\omega) = \frac{\left(i^{\frac{3}{2}}\alpha_0\right)(ber_1(\alpha_0)+ibei_1(\alpha_0))}{4(ber_2(\alpha_0)+ibei_2(\alpha_0))} \tag{24}$$

where $ber_1(\alpha_0), bei_1(\alpha_0), ber_2(\alpha_0), bei_2(\alpha_0)$ - the Thomson function for a given value $\alpha_0$can be calculated using special tables [19-23]. Having isolated the real and imaginary parts of the amplitude-phase frequency characteristic (24), we obtain

$$\chi = Re[\,W_{\tau_1,\vartheta_{x_1}}(i\omega)] = \frac{a_{\tau_1}}{a_{\vartheta_1}}\cos\phi_{\tau_1} = Re[\frac{\left(i^{\frac{3}{2}}\alpha_0\right)(ber_1(\alpha_0)+ibei_1(\alpha_0))}{4\left(ber_2(\alpha_0)+ibei_2(\alpha_0)\right)}] =$$
$$= \frac{\alpha_0}{4\sqrt{2}}\frac{(ber_1\alpha_0(bei_2\alpha_0-ber_2\alpha_0)-bei_1\alpha_0(bei_2\alpha_0+ber_2\alpha_0))}{(ber_2^2\alpha_0+bei_2^2\alpha_0)} \tag{25}$$

$$\beta = Im[\,W_{\tau_1,\vartheta_{x_1}}(i\omega)] = \frac{a_{\tau_1}}{a_{\vartheta_1}}\sin\phi_{\tau_1} = Im[\frac{\left(i^{\frac{3}{2}}\alpha_0\right)(ber_1(\alpha_0)+ibei_1(\alpha_0))}{4(ber_2(\alpha_0)+ibei_2(\alpha_0))}] =$$
$$=\frac{\alpha_0}{4\sqrt{2}}\frac{(ber_1\alpha_0(bei_2\alpha_0+ber_2\alpha_0)+bei_1\alpha_0(bei_2\alpha_0-ber_2\alpha_0))}{(ber_2^2\alpha_0+bei_2^2\alpha_0)} \tag{26}$$

Despite the fact that the Thomson functions are tabulated, calculations using formulas (25) and (26) lead to rather cumbersome calculations, and they cannot be used for oscillatory flow of a viscoelastic fluid in a cylindrical pipe. Therefore, if we assume that for large values the dimensionless frequencies of oscillations, that is $\left|i^{3/2}\alpha_0\right| > 1$, we expand the Bessel functions contained in (19) into asymptotic series, taking into account the formula [19].

$$i^{-n}J_n(ix) = \frac{e^x}{\sqrt{2\pi x}}[1 - \frac{4n^2-1}{1!(8x)} + \frac{(4n^2-1)(4n^2-3^2)}{2!(8x)^2} - \ldots]. \tag{27}$$

Substituting x its value instead $x = \sqrt{i}\alpha_0$ and keeping only three terms in the series under consideration with rounding of the coefficients at x, one can find an approximate expression (19) in the form

$$W_{\tau_1,\vartheta_{x_1}} = \frac{1}{4}(\frac{(x)J_1(ix))}{J_2(ix))}) = \frac{(x)}{4}(1 + \frac{12}{1!(8x)} + \frac{15\cdot 16}{2!(8x)^2}\cdots) \tag{28}$$

Considering that$x = \sqrt{i}\alpha_0$ in (28), we obtain

$$W_{\tau_1,\vartheta_{x_1}} = \frac{1}{4}(\frac{(i\sqrt{i}\alpha_0))J_1(i\sqrt{i}\alpha_0)}{J_2(i\sqrt{i}\alpha_0)}) = \frac{(\sqrt{i}\alpha_0)}{4}(1 + \frac{12}{1!(8\sqrt{i}\alpha_0)} + \frac{15\cdot 16}{2!(8\sqrt{i}\alpha_0)^2}\cdots) = \chi + \beta i \tag{29}$$

For oscillatory flow, from formula (29), having isolated its real and imaginary parts, we obtain

$$\chi = Re[\,W_{\tau_1,\vartheta_{x_1}}(i\omega)] = \frac{\alpha_0}{4\sqrt{2}} + \frac{3}{8} + \frac{15}{32\sqrt{2}\alpha_0} + \ldots,$$
$$\beta = Im[\,W_{\tau_1,\vartheta_{x_1}}(i\omega)] = \frac{\alpha_0}{4\sqrt{2}} - \frac{15}{32\sqrt{2}\alpha_0} + \cdots \tag{30}$$

Based on formula (22), taking into account formulas (25), (26) and (30), graphs are constructed in Fig. 1, showing the change in the relative shear stress in a non-stationary flow depending on the dimensionless oscillation frequency.

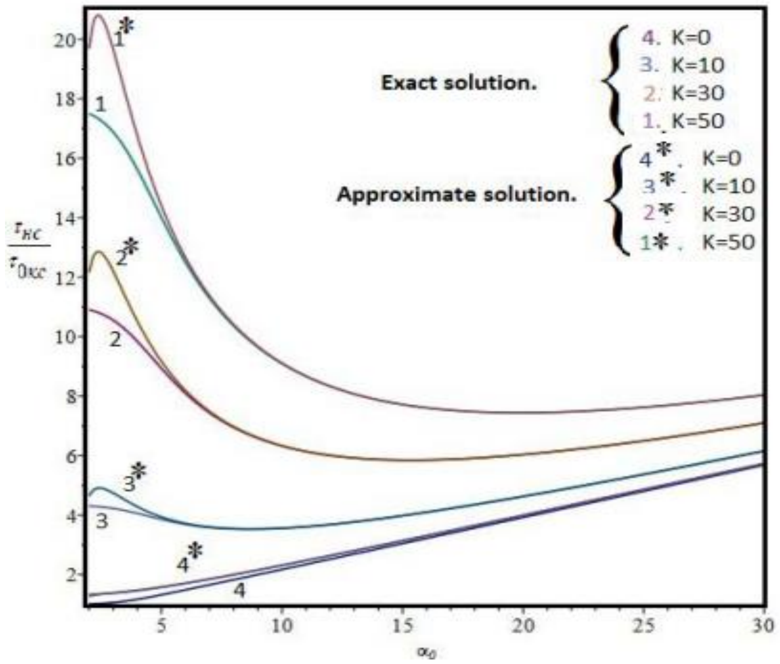

Fig. 1

From these graphs it is evident that the change in the relative shear stress in a non-stationary flow depending on the dimensionless oscillation frequency, at $\alpha_0 = 2$, differs by no more than 2% from its exact values, and at $\alpha_0 > 2$ these the difference approaches zero. Thus, in this case, approximate asymptotic formulas (30) can be used with high accuracy, starting with $\alpha_0 = 2$ for studying the oscillatory flow of a viscous and viscoelastic fluid in a cylindrical pipe.

4. Conclusion

Problems of oscillatory flow of a viscous incompressible fluid in a cylindrical pipe with a given harmonic oscillation of the average velocity across the pipe cross-section are solved. The transfer function and amplitude-phase frequency characteristics are determined. Using these functions, the influence of oscillation frequency and acceleration on the ratio of shear stress to cross-sectional velocity is determined. Asymptotic formulas are found for determining the transfer function for viscous fluid flow in unsteady flow at lower and higher oscillation frequencies. By comparing the exact solution to the asymptotic solution, new asymptotic formulas are proposed that are of great importance in studying the oscillatory flow of viscoelastic fluid in a cylindrical pipe.

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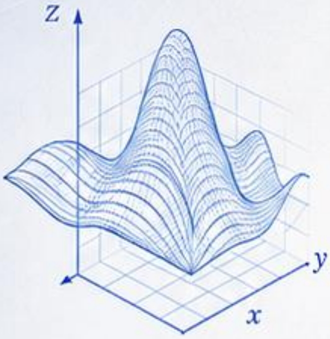


$$\frac{dx}{dt} = Ax + b(t)$$

$$\frac{\partial u}{\partial t} = \alpha \nabla^2 u + f$$

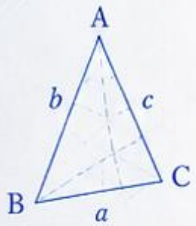
$$\sum_{i=1}^{n} a_i x_i + b$$

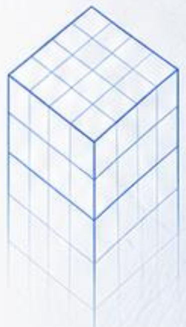
$$f(x) = \int_a^b f(x)\, dx$$

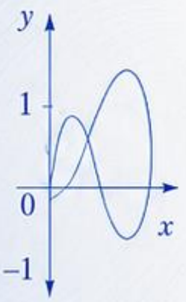
CONTENTS

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

$$\sum_{i=1}^{n} w_i x_i$$







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