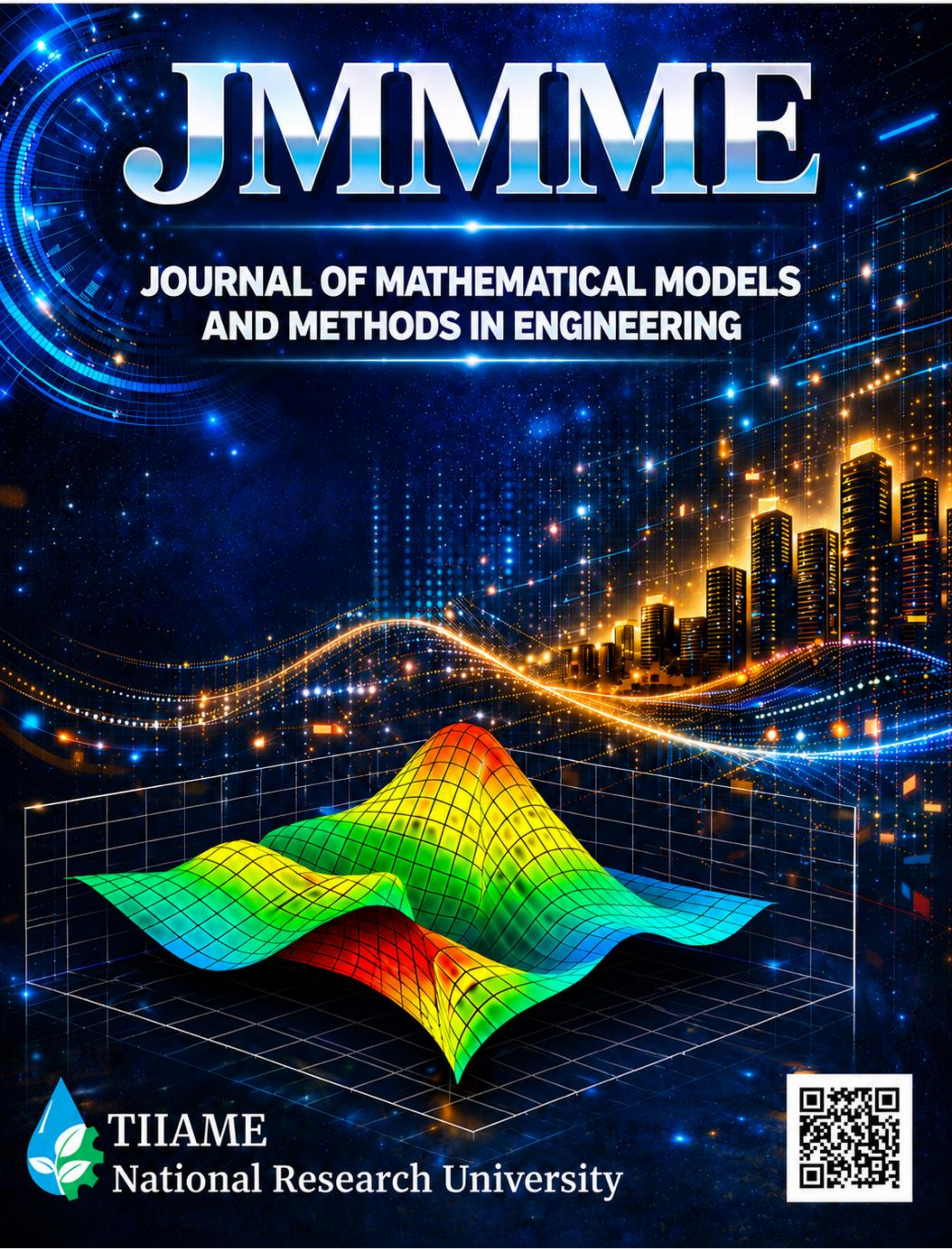
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Mathematical description and analysis of oscillatory processes of thin-walled structures when they are flown by a gas stream

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ABSTRACT

The flutter of viscoelastic three-layer Elements of an Aircraft, flown around by a supersonic gas flow is studied in the article. Mathematical models of problems on the flutter of viscoelastic three-layer plates with a structure asymmetrical in thickness, flown around by a supersonic gas flow are developed. The flutter of viscoelastic three-layer plates is studied in linear formulations. The critical flutter velocities of elongated plates are compared with the results obtained in previously published studies, where the solutions were obtained in an elastic formulation. The flutter of viscoelastic three-layer plates with a rigid filler that resists transverse shear, flown from the outside by a supersonic flow, was studied. It is shown that an increase in the geometric parameter characterizing the flexural rigidity of the bearing layers of three-layer structures leads to an increase in the flutter velocity by 25–40%.

1. Introduction

Layered structures, in particular, three-layer plates and shells, are used in the aircraft industry and in other branches of mechanical engineering, and in the future, the scope of their application will undoubtedly expand. Three-layer structures have many qualities that conventional structures made of metal do not have. They have high specific rigidity and can withstand high specific loads. Layered plates and shells have good heat and sound insulation properties, damping and vibration-absorbing properties, certain radio technical characteristics, etc. Due to the fact that there are no fasteners of any kind on the outer surface of three-layer structures and the surface is perfectly smooth, they have high aerodynamic qualities.

Theoretical and experimental studies on three-layer structures made it possible to reveal their main advantages in relation to other types of structures. These advantages are due to the fact that core layers reinforced with filler can withstand high compressive stresses. As a result, these designs are optimal in bending and can significantly increase their critical loads at a minimal weight. Their implementation in various branches of technology entailed intensive research in the field of theory and calculation methods. As a result, over the past fifty years, a separate direction has developed in the mechanics of a deformable rigid body related to the development of the theory of multilayer (in particular, three-layer) plates and shells [1–5].

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A great contribution to the development of the theory of multilayer plates and shells was made by E.I. Grigolyuk [3] and others. In [6], the main relations of the quadratic nonlinear theory of non-sloping multilayer anisotropic shells of an arbitrary form were presented.

In recent years, the needs of engineering have led to the fact that the bearing layers began to be made in different thicknesses. This greatly expands the scope of the use of three-layer structures.

Three-layer structures are effectively used in the aircraft industry and other branches of engineering. Therefore, three-layer structures have entailed intensive research in the field of flutter theory.

A number of publications [7, 8] are devoted to the issue of calculating vibrations of elastic three-layer plates and shells with a structure that is asymmetric in thickness. In the studies by A.I. Smirnov [7, 8], the supersonic flutter of three-layer plates and cylindrical shells was considered in an elastic formulation.

A mathematical model to investigate the effect of external average flow on sound transmission loss through composite sandwich panels filled with porous materials was built [9]. The equations of motion for multilayer composite plates were constructed using first-order shear deformation theory. A study of the influence of the external average flow, the angle of incidence, and the lamination scheme on the coincidence frequency and sound absorption of multilayer composite structures was conducted.

In [10], the study of the influence of free vibration on the behavior of a multifunctional sandwich plate using an advanced and reliable method was performed. The basic equation for the analysis of free vibrations was obtained, in which Reddy's third-order shear deformation theory was used.

In this article, the authors study the nonlinear flutter of viscoelastic three-layer plates and shallow shells with an asymmetric in thickness structure, which are flown around by a supersonic gas flow.

2.Equations of motion

Let us consider a rectangular three-layer plate with sides *a* and *b*, streamlined from the outside by a supersonic gas flow at an unperturbed velocity *V*, directed along the *Ox*-axis. The aerodynamic pressure is taken into account according to the linear piston theory [11]. Let us assume that the plates are hinged on all four edges.

The equation of motion of a viscoelastic three-layer plate in a gas flow in the absence of shear forces takes the following form:

$$\begin{aligned}&D(1-R^*)(1-\Theta h^2\beta_3^{-1}\nabla^2)\nabla^4\chi - N_x\frac{\partial^2}{\partial x^2}(1-h^2\beta_3^{-1}\nabla^2)\chi - \\ &- N_y\frac{\partial^2}{\partial y^2}(1-h^2\beta_3^{-1}\nabla^2)\chi - \Omega\frac{\partial^2}{\partial t^2}(1-h^2\beta_3^{-1}\nabla^2)\chi - q = 0.\end{aligned} \tag{1}$$

Here $\chi(x,y,t)$ is the displacement function related to the deflection $w(x, y, t)$ by the following relation:

$$w = (1-h^2\beta_3^{-1}\nabla^2)\chi, \tag{2}$$

where $\nabla^2=\partial^2/\partial x^2+\partial^2/\partial y^2$.

Values of D, Θ, β_3, Ω characterize the cylindrical stiffness of the three-layer package, the bending stiffness of the bearing layers, the shear stiffness of the filler, and the specific mass of the three-layer package, respectively; h is the package thickness; N_x, N_y are the external compressive (tensile) forces in the longitudinal and transverse directions; $q(x,y,t)$ is an aerodynamic load.

3.Numerical results

An approximate solution to equation (41) is sought in the following form:

$$\chi(x,y,t)=\sum_{n=1}^{N}\sum_{m=1}^{M}\chi_{nm}(t)\,\varphi_{nm}(x,y)\,, \tag{3}$$

where functions $\varphi_{nm}(x,y)$ are chosen so that each term of the sum (3) satisfies the boundary conditions at the edges of the plate, and $\chi_{nm}(t)$ are some functions to be determined. Substituting (3) into equation (1) and applying the Bubnov-Galerkin method to this equation, we obtain a system of integrodifferential equations with respect to the coefficients (3). By introducing the following dimensionless parameters

$$\frac{x}{a},\quad \frac{y}{b},\quad \frac{\chi}{h},$$

and keeping the previous notation, we obtain

$$A_{kl}\,\ddot{\chi}_{kl}+B_{kl}\,\dot{\chi}_{kl}+\left[\left(1-R^{*}\right)C_{kl}+E_{kl}\right]\chi_{kl}-V_{*}\sum_{n=1}^{N}F_{k\ln m}\chi_{nl}=0\,. \tag{4}$$

Here A_{kl}, B_{kl}, C_{kl}, E_{kl}, F_{klnm}, , $V_{*}=\aleph p_{\infty}a^{3}M^{*}/D$ are dimensionless parameters.

The calculation results are presented in Tables 1, 2.

4. Discussion of results

From Table 1 it can be seen that an increase in the viscosity coefficient A leads to a decrease in the critical flutter velocity $V_{*кр}$ by 59% [12-16]. For A=0 and A=0.1, the flutter velocity is 36 and 14.65, respectively. As seen from the table, the result obtained for the elastic plate (A=0) exactly coincides with the results given in [17].

The effect of external compressive (tensile) forces in the longitudinal and transverse directions was studied. Table 1 shows that an increase in compressive forces n_x ($n_x=N_x a^2/D$) in the direction of the flow velocity leads to a decrease in the critical flutter velocity. On the contrary, tensile forces n_x lead to the same proportional increase in the critical flutter velocity.

When the forces n_y ($n_y=N_y a^2/D$) change in the direction normal to the velocity of the flow V oncoming on the plates, the flutter velocity changes little.

An increase in parameter k_1 ($k_1=h^2\beta_3^{-1}/a^2$) leads to a significant change in V_{*cr}. The studies were conducted for k_1=0.1; 0.2; 0.5 and 1.5. It can be seen that with a decrease in the shear stiffness of the filler (an increase in coefficient κ_1) the critical flutter velocity of the three-layer plate decreases.

With an increase in the plate elongation λ ($\lambda=a/b$), the plate length in the flow direction increases and the elongated edges of the plate approach each other. The latter contributes to an increase in the relative rigidity of the system and an increase in the critical flutter velocity, which is observed from the table 2.

The influence of parameter ε (aerodynamic damping) was also studied. With an increase in coefficient ε, an increase in the dimensionless critical flutter velocity was observed.

Table 1

Critical values of flutter velocity depending on on the viscoelastic properties of the material

A	α	β	$-n_x$	$-n_y$	k_1	λ	Θ	ε	V_{*cr}
0									**36**
0.001	0.25	0.05	0.75	0.45	1	1	0.05	0.1	**34.2**
0.01									**19.45**
0.1									**14.65**
	0.1								**18.17**
0.01	0.5	0.05	0.75	0.45	1	1	0.05	0.1	**20**
	0.7								**21**
		0.01							**19.5**
0.01	0.25	0.08	0.75	0.45	1	1	0.05	0.1	**19.43**
		0.1							**19.42**

Table 2

Critical values of flutter velocity depending on the physical-mechanical and geometric nature of the plate

A	α	β	$-n_x$	$-n_y$	k_1	λ	Θ	ε	V_{*cr}
			3						**6.38**
			1.5						**15.1**
0.01	0.25	0.05	1	0,45	1	1	0.05	0.1	**17.93**
			0						**23.77**
			-0.5						**26.5**
			-1						**29.6**
			0.75	2.75					**17.15**
				0.5					**19.4**
0.01	0.25	0.05		0	1	1	0.05	0.1	**19.9**
				-0.5					**20.35**
				-1.5					**21.3**

				-4					**23.7**
					0.1				**70.5**
0.01	0.25	0.05	0.75	0.45	0.2	1	0.05	0.1	**45.5**
					0.5				**26.4**
					1.5				**16.99**
						1.2			**17.67**
0.1	0.25	0.05	0.75	0.45	1	1.5	0.05	0.1	**23.5**
						2			**36.82**
							0.03		**12.83**
0.01	0.25	0.05	0.75	0.45	1	1	0.07	0.1	**25.7**
								0	**19.4**
0.01	0.25	0.05	0.75	0.45	1	1	0.05	0.5	**19.6**
								5	**21.82**

5. Conclusions

1. Mathematical models of flutter problems for viscoelastic three-layer plates with a structure asymmetric in thickness, flown around by a supersonic gas flow, were constructed and developed.

2. A computational algorithm was developed based on the elimination of singularities of integrodifferential equations with singular kernels, followed by the use of quadrature formulas, for solving practically important linear and nonlinear problems of the hereditary theory of viscoelasticity.

3. When modeling flutter problems for viscoelastic three-layer plates, a number of new effects were obtained:

- it was determined that an account for the viscoelastic properties of the material of the three-layer elements of the aircraft leads to a decrease in the critical flutter velocity by 30 - 60%;
- it was found that an increase in compressive forces in the direction of the flow velocity leads to a decrease in the critical flutter velocity of three-layer plates. On the contrary, tensile forces lead to the same proportional increase in the critical flutter velocity;
- it was shown that an increase in the geometric parameter characterizing the flexural rigidity of the bearing layers of three-layer structures leads to an increase in the flutter velocity by 25–40%;
- it was determined that a decrease in the shear stiffness of the filler leads to a decrease in the flutter velocity.

References

[1] E.I. Grigolyuk, Finite deflections of three-layer shells with rigid filler, News of the AS USSR, OTN 1 (1958) 26–34.

[2] E.I. Grigolyuk, P.P. Chulkov, On the general theory of three-layer shells with a large deflection, Reports of the AS USSR, 150(5) (1963) 1012–1014.

[3] E.I. Grigolyuk, P.P. Chulkov, Stability and vibrations of three-layer shells. Moscow, "Engineering", 1973.

[4] V.A. Ivanov, V.N. Paimushin, Refined statement of dynamic problems of three-layer shells with transversally rigid filler and numerical-analytical method for their solution, Applied Mechanics and Technical Physics 36(4) (1995) 137-151.

[5] V.P. Paimushin, Theory of Stability of Three-Layer Plates and Shells (Stages of Development, State of the Art and Directions for Further Research), Proceedings of the Russian Academy of Sciences. Mechanics of a rigid body, 2 (1998) 148–162.

[6] E.I. Grigolyuk, V.I. Mamai, Nonlinear deformation of thin-walled structures, Moscow, Nauka. Fizmatlit, 1997.

[7] A.I. Smirnov, Dynamic stability and vibrations of three-layer panels in a supersonic gas flow, Reports of AS USSR, 180 (5) (1968) 1060–1063.

[8] A.I. Smirnov, Natural oscillations and flutter of three-layer cylindrical shells in a supersonic gas flow, Reports of AS USSR, 186(3) (1969) 533–536.

[9] Rasool Moradi-Dastjerdi, Kamran Behdinan, Free vibration response of smart sandwich plates with porous CNT-reinforced and piezoelectric layers, Applied Mathematical Modelling 96 (2021) 66-79. https://doi.org/10.1016/j.apm.2021.03.013.

[10] E.I. Grigolyuk, P.P. Chulkov, Critical loads of three-layer cylindrical and conical shells. Novosibirsk, Western-Siberian publishing house, 1966.

[11] A.A. Ilyushin, The law of plane sections in aerodynamics of high supersonic speeds, Applied Mathematics and Mechanics, XX (6) (1956) 733–755.

[12] B.A. Khudayarov, F.Z. Turaev, Nonlinear supersonic flutter for the viscoelastic orthotropic cylindrical shells in supersonic flow. Aerospace Science and Technology 84 (2019) 120-130.

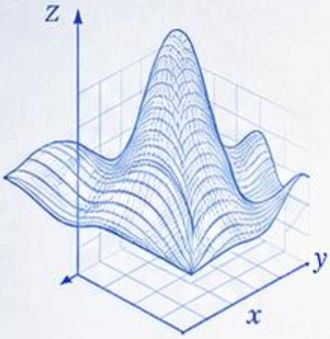
[13] B.A. Khudayarov, F.Z. Turaev, Nonlinear vibrations of fluid transporting pipelines on a viscoelastic foundation. Magazine of Civil Engineering 86(2) (2019) 30-45.

[14] B.A. Khudayarov, Modeling of supersonic nonlinear flutter of plates on a viscoelastic foundation. Advances in Aircraft and Spacecraft Science 6(3) (2019) 257-272.

[15] B.A. Khudayarov, F.Z. Turaev, Mathematical Simulation of Nonlinear Oscillations of Viscoelastic Pipelines Conveying Fluid. Applied Mathematical Modelling 66 (2019) 662-679.

[16] B.A. Khudayarov, Kh.M. Komilova, Vibration and dynamic stability of composite pipelines conveying a two-phase fluid flows. Engineering Failure Analysis 104 (2019) 500-512.

[17] A.I. Smirnov, Supersonic flutter of three-layer plates, Reports of the Academy of Sciences of the USSR, 183(3) (1968) 540–543.



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$$\frac{dx}{dt} = Ax + b(t)$$

$$\frac{\partial u}{\partial t} = \alpha \nabla^2 u + f$$

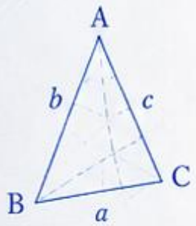
$$\sum_{i=1}^{n} a_i x_i + b$$

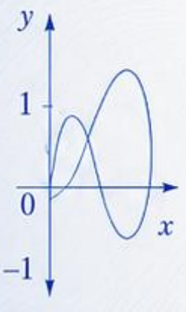
$$f(x) = \int_a^b f(x)\, dx$$

CONTENTS

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

$$\sum_{i=1}^{n} w_i x_i$$





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